Study Guide on Basic Reinsurance Pricing for the Society of Actuaries (SOA) Exam GIADV: Advanced Topics in General Insurance

(Based on David R. Clark's Paper, *"Basics of Reinsurance Pricing")*

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Source: Clark, David R., "Basics of Reinsurance Pricing", Actuarial Study Note, Revised 2014.

Problem BRP-1. According to Clark (p. 2), what is a major difference between reinsurance and primary insurance?

Solution BRP-1. A major difference is that a reinsurance program is tailored more closely to the buyer, so there does not exist an "average" reinsured or "average" reinsurance price (Clark, p. 2).

Problem BRP-2. What is the paradox of reinsurance pricing described by Clark (p. 2)?

Solution BRP-2. The paradox of reinsurance pricing is that a ceding company will not want to buy a reinsurance contract that can be precisely priced (Clark , p.2).

Problem BRP-3. How does surplus-share reinsurance differ from excess-of-loss reinsurance? Give a numerical example for each type of reinsurance. (Clark, p. 3)

Solution BRP-3. For surplus-share reinsurance, the retained line determines the reinsurer's proportional share of the risk. For instance, if the retained line is $100,000 on an insured value of $500,000, then the reinsurer will share in ($500,000 - $100,000)/$500,000 = 80% of *any* loss, no matter the amount of the loss. By contrast, an excess-of-loss reinsurance agreement would involve a fixed retention below which the ceding insurer absorbs all losses, and the reinsurer's obligation to pay is only triggered if the total loss amount exceeds the retention. For instance, in a treaty for $400,000 in excess of $100,000, the reinsurer will only pay for losses of size X in excess of $100,000 and would pay an amount of (X - $100,000), up to $400,000 in total.

Problem BRP-4. For a 6-line surplus-share reinsurance treaty with a retained line of $150,000, how much will the reinsurer pay for a $500,000 loss on an insured value of $750,000?

Solution BRP-4. The retained line is $150,000 and the reinsured portion could be up to $150,000*6 = $900,000. For an insured value of $750,000, the reinsured portion will be $750,000 - $150,000 = $600,000, so the proportional share of the reinsurer's involvement in any loss would be $600,000/$750,000 = 80%. The reinsurer would thus pay 80% of the $500,000 loss, or **$400,000.**

Problem BRP-5. For a 6-line surplus-share reinsurance treaty with a retained line of $150,000, how much will the reinsurer pay for a $500,000 loss on an insured value of $1,600,000?

Solution BRP-5. The retained line is $150,000 and the reinsured portion could be up to $150,000*6 = $900,000. For an insured value of $1,600,000, the full reinsurance portion of $900,000 will be utilized, since $1,600,000 - $150,000 = $1,450,000 > $900,000. The proportional share of the reinsurer's involvement in any loss would be $900,000/$1,600,000 = 56.25%. The reinsurer would thus pay 56.25% of the $500,000 loss, or **$281,250.**

Problem BRP-6. Clark, on p. 3, mentions proportional treaties that include fixed and variable quota-share arrangements on excess business. Describe the essence of such arrangements.

Solution BRP-6. The underlying business is excess-of-loss, but the reinsurer takes a proportional share of the ceding company's book of business. (Clark, p. 3)

Problem BRP-7. According to Clark (pp. 4-6), what are the six main steps for pricing a proportional reinsurance treaty?

Solution BRP-7. The six main steps for pricing a proportional reinsurance treaty are as follows:
1. Compile the historical experience on the treaty.
2. Exclude catastrophe and shock losses.
3. Adjust experience to ultimate level and project to the future period.
4. Select the expected non-catastrophe loss ratio for the treaty.
5. Load the expected non-catastrophe loss ratio for catastrophes.
6. Estimate the combined ratio, given the ceding commission and other expenses.

Problem BRP-8. When one is compiling the historical experience on a proportional surplus-share reinsurance treaty, and historical experience pertaining to premiums and incurred losses for the treaty is not available for five or more years, what adjustment should be made? (Clark, p. 4)

Solution BRP-8. The gross experience (prior to the reinsurance treaty) should be adjusted as if the surplus-share terms had been in place. This produces hypothetical treaty experience over the desired historical time period. (Clark, p. 4)

Problem BRP-9. Fill in the blanks:
(a) If a reinsurance treaty is written on a "risks attaching" basis, use ______ (type of premium) and _______ (type of losses).
(b) If a reinsurance treaty is written on a "losses occurring" basis, use ______ (type of premium) and _______ (type of losses).

Solution BRP-9. This question is based on the discussion by Clark, p. 4.
(a) If a reinsurance treaty is written on a "risks attaching" basis, use **written premium** and **losses covered by the attaching policies**.
(b) If a reinsurance treaty is written on a "losses occurring" basis, use **earned premium** and **accident-year losses.**

Problem BRP-10. Clark, on p. 4, discusses the second step of pricing for proportional reinsurance treaty – the exclusion of catastrophe and shock losses.

(a) Explain the distinction between catastrophe losses and shock losses.
(b) On what basis is each type of loss generally defined for property contracts?
(c) Describe some typical catastrophe losses and shock losses for casualty contracts.

Solution BRP-10.

(a) Catastrophe losses are due to a single event, such as an earthquake or hurricane, which may affect a large number of risks.

Shock losses are other losses, typically affecting a single policy, which may distort the overall results. (Clark, p. 4)

(b) For property contracts, **catastrophe losses** are generally defined on a per-occurrence (multiple-risk) basis. **Shock losses** are defined as large losses due to a single risk (single-risk basis). (Clark, p. 4)

(c) For casualty contracts, a typical **catastrophe loss** would be a claim impacting many insureds, such as an environmental liability claim. A typical **shock loss** would be a large settlement on a single policy. (Clark, p. 4)

Problem BRP-11. Clark, on pp. 4-5, discusses the third step of pricing for proportional reinsurance treaty – adjusting the experience to its ultimate level and projecting to the future time period. He notes that data from other sources may need to be used if treaty experience is insufficient to estimate loss-development factors. Clark then mentions two adjustments that might need to be made to these externally sourced factors. What are these possible adjustments?

Solution BRP-11.
Possible Adjustment 1. Adjust for the reporting lag to the reinsurer.
Possible Adjustment 2. Adjust for accident-year/policy-year differences.

Problem BRP-12. Clark, on pp. 4-5, discusses the third step of pricing for proportional reinsurance treaty – adjusting the experience to its ultimate level and projecting to the future time period.

(a) When adjusting historical premiums to the future level, what is the starting point, according to Clark?
(b) For "losses occurring" reinsurance treaties, what method can be used to calculate rate-level adjustment factors?
(c) Which rate changes' impact has to be included in the calculation of rate-level adjustment factors? Why would this be an area requiring judgment?

Solution BRP-12.
(a) According to Clark (p. 4), the starting point when adjusting historical premiums to the future level is **historical changes in rates and average pricing factors** (such as schedule-rating credits).
(b) For "losses occurring" reinsurance treaties, the **parallelogram method** can be used to calculate rate-level adjustment factors.
(c) The impact of **rate changes anticipated during the treaty period** needs to be considered. This area requires judgment because some of these changes might not have been filed and/or approved at the time of the calculation. (Clark, p. 5)

Problem BRP-13. Clark, on pp. 4-5, discusses the third step of pricing for proportional reinsurance treaty – adjusting the experience to its ultimate level and projecting to the future time period.
(a) If the premium base for a property exposure is insured value, what adjustment factor should be included for historical premium?
(b) What is one possible source for information to use for loss-trend adjustments?

Solution BRP-13.
(a) An **exposure inflation factor** should be included in the adjustment for historical premium.
(b) The **ceding company's rate filings** are a possible source for information to use for loss-trend adjustments.

Problem BRP-14. ***Fill in the blanks*** (Clark, p. 5): In selecting the expected non-catastrophe loss ratio for a proportional reinsurance treaty, if the adjusted historical experience data are reliable, the expected loss ratio is simply equal to the average of the _____________, adjusted to the _________. It is recommended to compare this to the ____________, available in the ceding company's Annual Statement, and to __________.

Solution BRP-14. In selecting the expected non-catastrophe loss ratio for a proportional reinsurance treaty, if the adjusted historical experience data are reliable, the expected loss ratio is simply equal to the average of the **historical loss ratios**, adjusted to the **future level**. It is recommended to compare this to the **ceding company's gross calendar-year experience**, available in the ceding company's Annual Statement, and to **industry averages**. (Clark, p. 5)

Problem BRP-15.
(a) Clark (p. 5) states that, typically, there will be insufficiently credible historical loss experience to price *what* loading for a proportional reinsurance treaty?
(b) What method have reinsurers used to price this loading in the past?
(c) For what types of loss events might the method in part (b) still be used today?
(d) What is the most common current procedure for selecting this loading for types of loss events for which the method in part (b) is *not* used?

Solution BRP-15.
(a) Typically, there will be insufficiently credible historical loss experience to price a loading **for catastrophe potential.**
(b) In the past, reinsurers have used a method of spreading large losses over expected payback periods. A 1-in-n-year event would correspond to a loading of (100/n)% of the loss amount.
(c) This payback approach might still be used for **casualty** loss events.
(d) The most common current procedure for selecting a catastrophe loading for property loss events is to rely on an engineering-based catastrophe model that incorporates the risk profile of the ceding company. (Clark, p. 5)

Problem BRP-16. Identify three "other" features of a proportional reinsurance treaty that must be evaluated (if they exist) when estimating the treaty's combined ratio *after* the total expected loss ratio is estimated. (Clark, p. 5)

Solution BRP-16.
1. Ceding commission (often a sliding-scale commission)
2. Reinsurer's general expenses and overhead
3. Brokerage fees

Problem BRP-17. Clark (p. 6) mentions two elements that should be taken into account in the evaluation of reinsurance treaty terms to determine whether the projected combined ratio on the treaty is acceptable. What are these two elements?

Solution BRP-17.
1. Potential investment income
2. Risk level of the exposures

Problem BRP-18.
(a) According to Clark (p. 9), after an expected loss ratio is estimated for a proportional reinsurance treaty, what will often remain as areas of disagreement between the ceding company and the reinsurer?
(b) What is often built into proportional reinsurance treaties to resolve situations where the reinsurer might now "follow the fortunes" of the ceding company?

Solution BRP-18.
(a) There will often remain disagreement about the **loss ratio** and the **appropriate ceding commission.**
(b) Adjustable features are often built into proportional reinsurance treaties to resolve such situations. (Clark, p. 9)

Problem BRP-19. In situations where there is a sliding-scale ceding commission that varies on the basis of loss ratio, Clark (pp. 10-11) distinguishes between a "naïve" approach of estimating the expected ceding commission and a more accurate approach. Conceptually describe each and explain why the latter approach is more accurate.

Solution BRP-19. The "naïve" approach works as follows:
1. Calculate the expected loss ratio.
2. The expected ceding commission is the ceding commission corresponding to the expected loss ratio.

The improved approach works as follows:
1. Calculate the ceding commission associated with the average loss ratio for each range in an aggregate loss distribution model.
2. Calculate a probability-weighted average ceding commission, based on the ceding commissions in each range of the aggregate loss distribution model.

The second approach is preferable because it takes into account the possibility (indeed, the likelihood) that the distribution of ceding commissions may differ from the distribution of loss ratios (i.e., a ceding commission may "slide" differently depending on the range of loss ratios being considered). Thus, simply calculating an expected loss ratio fails to capture the impact of loss-ratio variation on the ceding commission. Using an aggregate loss distribution at least enables one to account for the differential "slides" of the ceding commission within each selected loss ratio range.

Problem BRP-20.
(a) When ceding commissions for a proportional reinsurance treaty can vary on a sliding scale, what are two shortcomings of the approach of estimating an expected commission based on the historical loss ratios, adjusted to a future level and including catastrophe and shock losses?

(b) What kind of model can remedy these shortcomings? (Clark, p. 10)

Solution BRP-20.
(a) Any two of the answers below are acceptable:
Shortcoming 1. The calculation may be distorted by historical catastrophes.
Shortcoming 2. The calculation may be distorted by years with low premium volume.
Shortcoming 3. The calculation leaves out many possible outcomes. (Clark, p. 10)

(b) An **aggregate loss distribution model** can remedy these shortcomings.

Problem BRP-21. What is a *carryforward provision* for a ceding commission? Give a brief numerical example of how such a provision would work. (See Clark, p. 11)

Solution BRP-21. A carryforward provision is a clause in the reinsurance contract that allows subsequent years' ceding commissions to be modified by any of the primary insurer's prior-year loss amounts in excess of the loss ratio corresponding to the minimum ceding commission.
For instance, if the minimum ceding commission is 10%, corresponding to an 80% loss ratio, and the ceding company's loss ratio in Year X is 85%, then the loss amount corresponding to the excess 5% of losses may be used in calculating the Year (X+1) loss ratio for ceding-commission purposes.

Problem BRP-22. Clark (pp. 11-13) describes two ways of addressing carryforward provisions in estimating expected ceding commissions for proportional reinsurance contracts. Conceptually describe each approach and identify a shortcoming of each.

Solution BRP-22. The following are two ways of addressing carryforward provisions in estimating expected ceding commissions for proportional reinsurance contracts:

1. Shift the ceding commission "slide" by the amount of the carryforward from prior years. For instance, if the ceding commission slides 1:1 from 10% at an 80% loss ratio to 30% at a 60% loss ratio, and the carryforward from prior years is +3%, then one could adjust the slide to 1:1 from 10% at an 77% loss ratio to 30% at a 57% loss ratio.
***Shortcoming*:** Only carryforward for the current year is addressed; the possibility of carryforward for subsequent years is not taken into account.

2. Look at the long-run circumstances of the contract and, instead of applying the sliding scale to just the current year, apply it to a longer block of years. This allows for reduced aggregate variance in losses.
Shortcomings (any one will suffice): (1) The approach assumes that the reinsurance contract will be renewed over many years - which is often not a certainty. (2) There is no way of addressing situations where only ceding commission deficits, but not credits, can be carried forward. (3) There is no unambiguous way of estimating the variation reduction achieved as a result of this approach.

Problem BRP-23. Provide a formula for the Reinsurer's Profit Commission (RPC), using the following variables:
• ALR = Actual Loss Ratio
• CC = Ceding Commission
• M = Margin for Expenses
• PR = Percent Returned (as percent of reinsurer's profit)
(Clark, p. 13)

Solution BRP-23. RPC = PR*(100% – ALR – CC – M)

Problem BRP-24. Suppose the following is known about a reinsurance treaty:
The actual loss ratio is 43%.
The ceding commission is 30%.
The reinsurer's margin for expenses from the treaty premium is 6%.
The percent returned from the profit to comprise the reinsurer's profit commission is 41%.

Calculate the Reinsurer's Profit Commission (RPC) as a percentage of premium.

Solution BRP-24. We use the formula RPC = PR*(100% –ALR – CC – M) = 41%*(100% - 43% - 30% - 6%) = 41%*21% = **RPC = 8.61%.**

Problem BRP-25. What two similarities does Clark (p. 13) note regarding the estimation of the reinsurer's profit commission and the sliding-scale ceding commission?

Solution BRP-25.
1. Each type of commission should be evaluated using an aggregate distribution on the loss ratio.
2. For both types of commission, there is some ambiguity regarding the handling of carryforward provisions. (Clark, p. 13)

Problem BRP-26. What is a *loss corridor*? Provide a numerical example of how this feature of a reinsurance contract would work. (Clark, p. 13)

Solution BRP-26. A loss corridor provides that the ceding company will reassume a portion of the reinsurer's liability if the loss ratio exceeds a certain amount.

For instance, there might be a loss corridor of 60% of the layer from a 75% to a 95% loss ratio. Suppose the reinsurer's loss ratio prior to the application of the corridor is 110%. Then, after the application of the corridor, the reinsurer's loss ratio would be 110%-60%*(95%-75%) = 98%.

Problem BRP-27. Suppose a reinsurance treaty involves a loss corridor of 36% between a 50% and an 80% loss ratio. Before the application of the corridor, the reinsurer's loss ratio is 97%. What is the reinsurer's loss ratio after the application of the corridor?

Solution BRP-27. We consider situation before and after the application of the corridor.

	Before Corridor	**After Corridor**	**Calculation**
Loss Ratio Below Corridor	50%	50%	Capped at 50%
Loss Ratio Within Corridor	30%	19.2%	30%-36%*(80%-50%)
Loss Ratio Above Corridor	17%	17%	97% - 80%
TOTAL LOSS RATIO	**97%**	**86.2%**	

Thus, the reinsurer's loss ratio after the application of the corridor is **86.2%.**

Problem BRP-28. Suppose a reinsurance treaty involves a loss corridor of 36% between a 50% and an 80% loss ratio.

You also have the following analysis of loss ratios *prior* to the application of the corridor, using an aggregate distribution:

Range of Loss Ratios	**Average Loss Ratio in Range**	**Probability of Being in Range**
0%-50%	43%	0.344
50%-80%	72%	0.366
80% or above	101%	0.290
ALL	70.434%	1.000

Calculate the overall average loss ratio using this aggregate loss distribution, *after* the application of the loss corridor.

Solution BRP-28. We calculate as follows:

Range of Loss Ratios	Average Loss Ratio in Range *After* Application of Corridor (Net of Corridor)	Probability of Being in Range
0%-50%	43% (Does not change)	0.344
50%-80%	72% - 36%*(72%-50%) = 64.08%	0.366
80% or above	101% - 36%*(80%-50%) = 90.2%	0.290
ALL	0.344*43% + 0.366*64.08% + 0.290*90.2% = **64.40328%**	1.000

Thus, the loss corridor has reduced the reinsurer's loss ratio to **64.40328%.**

Problem BRP-29. Explain how property per-risk excess reinsurance treaties provide narrower coverage than property per-occurrence excess treaties. (Clark, p. 14)

Solution BRP-29. Property per-risk excess treaties typically apply the coverage layer to a single property location, while per-occurrence treaties apply to multiple risks to provide catastrophe protection. (Clark, p. 14)

If a large loss applies to multiple risks, it is more likely that the retention will be exceeded in aggregate for the per-occurrence treaty, than for the per-risk treaty, where it would need to be exceeded for each risk separately.

Problem BRP-30. What is the meaning of the oxymoronic terms "gross net earned premium income" and "gross net written premium income" – which are often used to describe the subject premium for a property per-risk excess reinsurance treaty? (Clark, p. 14)

Solution BRP-30. The premium income is net of any other reinsurance inuring to the benefit of the per-risk treaty, but gross of the per-risk treaty being priced. (Clark, p. 14)

Problem BRP-31. Identify and briefly describe the two major pricing approaches for property per-risk excess reinsurance treaties. (Clark, pp. 14, 17)

Solution BRP-31.
Approach 1: Experience rating – assumes that the historical experience, adjusted properly, is the best predictor of future expectations.
Approach 2: Exposure rating – models the current risk profile and centers on an exposure curve which represents the amount of loss capped at a given percent of the insured value relative to the total value of the loss. (Clark, p. 17)

Problem BRP-32. Identify the five steps described by Clark (pp. 14-15) for the experience-rating approach of pricing property per-risk excess reinsurance treaties.

Solution BRP-32.
Step 1. Gather the subject premium and historical losses for as many recent years as possible.
Step 2. Adjust the subject premium to the future level using rate, price, and exposure inflation factors.
Step 3. Apply loss inflation factors to the historical large losses and determine the amount included in the layer being analyzed.
Step 4. Apply excess development factors to the summed losses in each period. Apply frequency trend, if needed.
Step 5. Divide the trended and developed layer losses by the adjusted subject premium to obtain loss costs by year, which may be averaged to project the expected loss cost. (Clark, pp. 14-15)

Problem BRP-33. ***Fill in the blanks*** (Clark, p. 14): When gathering the subject premium and historical losses for recent years in pricing property per-risk excess reinsurance treaties, the number of years relied upon in the final analysis should be a balance between _______ and ________. The historical analysis should include all losses that would pierce the layer being priced after the application of ___________.

Solution BRP-33. When gathering the subject premium and historical losses for recent years in pricing property per-risk excess reinsurance treaties, the number of years relied upon in the final analysis should be a balance between **credibility** and **responsiveness**. The historical analysis should include all losses that would pierce the layer being priced after the application of **trend factors**. (Clark, p. 14)

Problem BRP-34. When using an experience-rating approach and summing the amount of losses that fall into the reinsured layer for each historical period, what should be done to allocated loss-adjustment expense (ALAE) if it applies *pro rata* with losses? (Clark, p. 15)

Solution BRP-34. If ALAE applies *pro rata* with losses, it should be added in individually for each loss. (Clark, p. 15)

Problem BRP-35. ***Fill in the blanks*** (Clark, p. 15): In any experience-rating model for reinsurance pricing, the loss-development factors should be derived from the same _______, if possible.

Solution BRP-35. In any experience-rating model for reinsurance pricing, the loss-development factors should be derived from the same **ceding-company data**, if possible. (Clark, p. 15)

Problem BRP-36. ***Fill in the blanks*** (Clark, p. 15): The projected loss costs resulting from an experience-rating analysis should be ___________ distributed about the average. If loss costs are _____________ or ______________, then assumptions of the model may need to be re-examined as the __________ or ___________ factors may be too high or low, or there may have been shifts in the _______________ or _______________ written by the company.

Solution BRP-36. The projected loss costs resulting from an experience-rating analysis should be **randomly** distributed about the average. If loss costs are **increasing** or **decreasing**, then assumptions of the model may need to be re-examined, as the **trend** or **development** factors may be too high or low, or there may have been shifts in the **types of business** or **sizes of risks** written by the company. (Clark, p. 15)

Problem BRP-37. You have the following loss and premium information regarding historical losses for a ceding company that seeks to purchase a property per-risk excess reinsurance treaty with an attachment point of 250,000 Golden Hexagons (GH) and a limit of 500,000 GH. The treaty is to take effect on 1/1/2329.

Accident Date	Untrended Total Loss	Number of Days from 1/1/2329
11/24/2325	163,000	1134
12/4/2325	350,000	1124
12/1/2326	265,000	762
3/4/2327	243,000	669
6/6/2327	666,000	575
7/7/2328	125,000	178
12/11/2328	890,000	21

Accident Year	On-Level Subject Premium	Loss-Development Factor (LDF)
2325	2,400,000	1.02
2326	2,200,000	1.05
2327	1,825,000	1.25
2328	3,333,000	1.65

When using an experience-rating approach, you assume an annual loss trend of 5%.

Use this experience-rating approach and all of the premium and loss information and adjustments given above to estimate a loss cost for the reinsurance treaty. Express your answer as a percentage, rounded to two decimal places.

Solution BRP-37.

The first step is to trend all historical losses to the level they would have been at, had they occurred on 1/1/2329. Let n be the number of days between the date of loss and 1/1/2329. The trending is done by applying a factor of $1.05^{n/365}$ to the untrended loss amount. Then we find the amount of trended loss within the reinsured layer.

(a) Accident Date	(b) Untrended Total Loss	(c) Number of Days from 1/1/2329 (n)	(d) $1.05^{n/365}$	(e) Trended Total Loss = (b)*(d)	(f) Amount of Trended Loss in Reinsured Layer (MAX(0, MIN(500,000, (e) – 250,000))
11/24/2325	163,000	1134	1.1636757	189,679.14	0
12/4/2325	350,000	1124	1.1621212	406,742.43	156,742.43
12/1/2326	265,000	762	1.107226	293,414.90	43,414.90
3/4/2327	243,000	669	1.0935468	265,731.87	15,731.87
6/6/2327	666,000	575	1.0798922	719,208.20	469,208.20
7/7/2328	125,000	178	1.0240789	128,009.86	0
12/11/2328	890,000	21	1.002811	892,501.83	500,000

Then we can add together the trended losses within the reinsurance layer, pertaining to each individual accident year.

(g) Accident Year	(h) On-Level Subject Premium	(i) LDF	(j) Trended Loss in Reinsured Layer	(k) Developed, Trended Loss in Reinsured Layer = (i)*(j)	(l) Loss Cost (as %) = (k)/(h)
2325	2,400,000	1.02	156,742.43	159,877.28	6.66%
2326	2,200,000	1.05	43,414.90	45,585.65	2.07%
2327	1,825,000	1.25	484,940.07	606,175.09	33.22%
2328	3,333,000	1.65	500,000	825,000.00	24.75%
TOTAL	**9,758,000**		**1,185,097.40**	**1,636,638.01**	**16.77%**

The total estimated loss cost using the historical data and adjustments is therefore **16.77%**.

Problem BRP-38. Let P be an exposure curve, which represents the amount of loss capped at a percent (p) of insured value (IV) relative to the total value of the loss.

Let f(x) be the probability density function of the individual loss amount, and let F(x) be the cumulative distribution function of this loss amount. Let $E(X) = {}_0^{\infty}\int x*f(x)dx$ be the expected value of the loss amount.

Provide the formula for P(p) in terms of the values defined above. (Clark, p. 17)

Solution BRP-38. $\mathbf{P(p) = {}_0^{p*IV}\int[1\text{-}F(x)]dx/[E(x)]}$

Problem BRP-39. For an exposure curve P, if IV = Insured Value, R = Retention, and L = Limit for a property per-risk excess reinsurance treaty, provide the expression for the portion of the expected loss on the risk which falls in the treaty layer. (Clark, p. 17)

Solution BRP-39. P((R+L)/IV) – P(R/IV).

Problem BRP-40. You are given data for a home insurance book of business, by type of loss.

Loss as a % of Insured Value	Cumulative Fire Loss Cost Distribution	Cumulative Wind Loss Cost Distribution	Cumulative "Other Perils" Loss Cost Distribution
10%	25%	63%	42%
20%	35%	68%	48%
30%	40%	77%	50%
40%	43%	82%	54%
50%	46%	85%	56%
60%	49%	89%	57%
70%	55%	92%	60%
80%	63%	94%	69%
90%	68%	97%	72%
100%	73%	98%	79%
110%	81%	99%	84%
120%	86%	100%	89%
130%	89%	100%	91%
140%	93%	100%	92%
150%	96%	100%	94%
160%	98%	100%	98%
170%	99%	100%	100%
180%	100%	100%	100%
190%	100%	100%	100%
200%	100%	100%	100%

Calculate the exposure factor – the percentage portion of the expected loss on the risk which falls in the treaty layer of coverage of $200,000 in excess of $100,000 – for the following insured values:

(a) Insured value of 100,000 for Fire coverage only
(b) Insured value of 200,000 for Wind coverage only
(c) Insured value of 400,000 for “Other Perils” coverage only
(d) Insured value of 500,000 for Fire coverage only

Use linear interpolation where necessary.

Solution BRP-40. For each part, we apply the formula P((R+L)/IV) – P(R/IV), with P being the appropriate exposure curve. We are given R = 100,000, L = 200,000, and we are given IV in each case.

(a) IV = 100,000, so P_{Fire}((R+L)/IV) – P_{Fire}(R/IV) = P_{Fire}(300,000/100,000) - P_{Fire}(100,000/100,000) = P_{Fire}(300%) - P_{Fire}(100%) = 100% - 73% = **27%.**

(b) IV = 200,000, so P_{Wind}((R+L)/IV) – P_{Wind}(R/IV) = P_{Wind}(300,000/200,000) – P_{Wind}(100,000/200,000) = P_{Wind}(150%) - P_{Wind}(50%) = 100% - 85% = **15%.**

(c) IV = 400,000, so P_{Other}((R+L)/IV) – P_{Other}(R/IV) = P_{Other}(300,000/400,000) - P_{Other}(100,000/400,000) = P_{Other}(75%) - P_{Other}(25%). We use linear interpolation:

P_{Other}(75%) = [P_{Other}(70%) + P_{Other}(80%)]/2 = (60% + 69%)/2 = 64.5%

P_{Other}(25%) = [P_{Other}(20%) + P_{Other}(30%)]/2 = (48% + 50%)/2 = 49%

Thus, P_{Other}(75%) - P_{Other}(25%) = 64.5% - 49% = **15.5%.**

(d) IV = 500,000, so P_{Fire}((R+L)/IV) – P_{Fire}(R/IV) = P_{Fire}(300,000/500,000) - P_{Fire}(100,000/500,000) = P_{Fire}(60%) - P_{Fire}(20%) = 49%-35% = **14%.**

Problem BRP-41. According to Clark (p. 18), what is the reason that exposure curves for pricing property per-risk excess reinsurance treaties allow for exposures above the insured value?

Solution BRP-41. Often the limits profile provided for the reinsured business does not include business interruption coverage for commercial policies or living expenses for home insurance policies. Therefore, the ceding company's actual obligation for a loss may exceed its provided policy limit. (Clark, p. 18)

Problem BRP-42. ***Fill in the blanks*** (Clark, p. 18): When analyzing the limits profile provided for an exposure-rating approach to price a property per-risk excess reinsurance treaty, one should question it to verify that the size-of-risk ranges are on a __________ basis. Distortions will result if the limits profile is assembled using total values for policies covering ___________.

Solution BRP-42. When analyzing the limits profile provided for an exposure-rating approach to price a property per-risk excess reinsurance treaty, one should question it to verify that the size-of-risk ranges are on a **per-location** basis. Distortions will result if the limits profile is assembled using total values for policies covering **multiple locations**. (Clark, p. 18)

Problem BRP-43.

(a) What assumption is implicit in the exposure-rating approach for pricing property per-risk reinsurance treaties?

(b) For what line of business might this assumption be appropriate, according to Clark?

(c) For what line of business might this assumption be a serious problem, according to Clark?

(Clark, p. 19)

Solution BRP-43.

(a) The implicit assumption is that the same exposure curve applies regardless of the size of the insured value.

(b) This assumption might be appropriate for **homeowners' insurance** business.

(c) This assumption may pose a serious problem for **property insurance of large commercial risks.**

Problem BRP-44. Discuss how the problem of "free cover" might arise for a property per-risk excess reinsurance treaty and provide a numerical example. (Clark, p. 20)

Solution BRP-44. If experience rating is used to price a layer of a reinsurance treaty for which there are no losses trending into the highest portion of the layer, then the indicated rate with and without coverage for that portion of the layer would be the same, falsely suggesting that there is no incremental risk in that layer.

For example, on a reinsurance treaty covering $900,000 in excess of $100,000, if the largest trended historical loss from the ground up is $600,000, then the experience-rating approach would generate the same indication for the treaty for $900,000 in excess of $100,000 as it would for a treaty for $500,000 in excess of $100,000.

Problem BRP-45. What is a possible approach suggested by Clark (p. 20) to solve the problem of "free cover" for a property per-risk excess reinsurance treaty?

Solution BRP-45. A possible solution is to use experience rating to price only the lowest portion of the treaty layer, and then to use exposure rating to project losses for the higher layer for which no historical data are available.

Problem BRP-46. According to Clark (p. 20), why is the actual number of claims during the historical period in question not a good basis for credibility of reinsurance data? What is a superior basis?

Solution BRP-46. The actual number of claims during the historical period in question not a good basis for credibility, because this measure implies that fortuitously worse time periods in terms of loss are necessarily assigned more credibility, thus making the reinsurer's experience appear worse than it actually is. The number of claims *expected* during the historical period is a better basis for credibility, because it does not bias the data by assigning more credibility to less favorable experience.

Problem BRP-47. What measure of credibility could be used if the expected number of claims for a property per-risk excess reinsurance treaty is not easily calculable? (Clark, p. 20)

Solution BRP-47. The **dollars of expected loss, based on the exposure rating**, might be used as an alternative measure of credibility if the expected number of claims is not easily calculable. (Clark, p. 20)

Problem BRP-48. ***Fill in the blanks*** (Clark, p. 20): As a measure of credibility when pricing a property per-risk excess reinsurance treaty, it is appropriate to look at the _________________ in the projected _________from each of the historical periods. Stability of this measure should _________ credibility even if the number of claims is relatively small.

Solution BRP-48. As a measure of credibility when pricing a property per-risk excess reinsurance treaty, it is appropriate to look at the **year-to-year variation** in the projected **loss cost** from each of the historical periods. Stability of this measure should **add** credibility even if the number of claims is relatively small.

Problem BRP-49. According to Clark (p. 21), when there exists inuring reinsurance and *experience rating* is used to price a property per-risk excess reinsurance treaty, what is the only accurate way to reflect the existence of underlying reinsurance that inures to the benefit of the treaty being priced?

Solution BRP-49. The only accurate way is to restate the historical loss experience on a basis net of the inuring reinsurance. (Clark, p. 21)

Problem BRP-50.
(a) According to Clark (p. 21), when there exists surplus-share reinsurance inuring to the benefit of a property per-risk excess treaty and *exposure rating* is used to price a property per-risk excess reinsurance treaty, the exposure rating can be applied directly to a risk profile, but how must the risk profile be adjusted?
(b) ***Fill in the blanks*** (Clark, p. 21): If the actuary has exposure curves varying by size of insured value, and there exists surplus-share reinsurance inuring to the benefit of a property per-risk excess treaty, the curve should be selected based on the insured value _________ (before or after?) the surplus share is applied, but the exposure factor should apply to the subject premium _________ (before or after?) the surplus share is applied.

Solution BRP-50.
(a) The risk profile must be adjusted to reflect the terms of the insuring surplus-share treaty.
(b) If the actuary has exposure curves varying by size of insured value, and there exists surplus-share reinsurance inuring to the benefit of a property per-risk excess treaty, the curve should be selected based on the insured value **before** the surplus share is applied, but the exposure factor should apply to the subject premium **after** the surplus share is applied. (Clark, p. 21)

Problem BRP-51. You are given the following exposure curve for a homeowners' insurance book of business.

Loss as a % of Insured Value	Cumulative Loss-Cost Distribution
10%	25%
20%	35%
30%	40%
40%	43%
50%	46%
60%	49%
70%	55%
80%	63%
90%	68%
100%	73%
110%	81%
120%	86%
130%	89%
140%	93%
150%	96%
160%	98%
170%	99%
180%	100%
190%	100%
200%	100%

The company's distribution of direct premium by insured value is also known.

Limit of Insurance	Direct Premium
25,000	244,404
50,000	124,444
100,000	324,019
200,000	120,310
400,000	44,000
500,000	58,000

(a) This primary insurer obtains a property per-risk excess reinsurance treaty. Find the exposure rate (equal to Exposure Premium/Direct Premium) for the layer of coverage of $200,000 in excess of $100,000, assuming there is no inuring reinsurance.

(b) Now assume that a surplus-share reinsurance treaty with a retained line of $300,000 inures to the benefit of the property per-risk excess reinsurance treaty. Find the revised exposure rate for the treaty. Use linear interpolation on the given exposure curve, where necessary.

(c) Given an expected loss ratio of 55% in both situations, calculate the expected losses for the property per-risk excess reinsurance treaty, both with and without the presence of the inuring surplus-share reinsurance treaty.

Solution BRP-51.

(a)

(a) Insured Value	**(b) Direct Premium**	**(c) Retention as % of Insured Value = 100,000/(a)**	**(d) Retention + Limit as % of Insured Value = 300,000/(a)**	**(e) Exposure Factor = P(d) – P(c), where P is the given exposure curve**	**(f) Exposure Premium = (b)*(e)**
25,000	244,404	400%	1200%	0%	0
50,000	124,444	200%	600%	0%	0
100,000	324,019	100%	300%	27%	87,485.13
200,000	120,310	50%	150%	50%	60,155
400,000	44,000	25%	75%	21.5%	9,460
500,000	58,000	20%	60%	14%	8,120

Exposure rate = [Total in (f)] / [Total in (b)] = 165,220.13/915,177 ≈ **18.05%.**

(b) We would need to re-state the subject premium net of the insuring reinsurance treaty.
For the insured value of 400,000, a retained line of 300,000 would mean that the net premium is P(75%) = (55% + 63%)/2 = 59% of the direct premium, or 59%*44000 = 25,960.
For the insured value of 500,000, a retained line of 300,000 would mean that the net premium is P(60%) = 49% of the direct premium, or 49%*58000 = 28,420.
Then we set the new net insured values for these original higher limits to 300,000 and apply the given exposure curve with respect to these adjusted insured values.

(a) Net Insured Value	**(b) Net Premium**	**(c) Retention as % of Insured Value = 100,000/(a)**	**(d) Retention + Limit as % of Insured Value = 300,000/(a)**	**(e) Exposure Factor = P(d) – P(c), where P is the given exposure curve**	**(f) Exposure Premium = (b)*(e)**
25,000	244,404	400%	1200%	0%	0
50,000	124,444	200%	600%	0%	0
100,000	324,019	100%	300%	27%	87,485.13
200,000	120,310	50%	150%	50%	60,155
300,000	25,960	33.333%	100%	32%	8,307.20
300,000	28,420	33.333%	100%	32%	9,094.40

To find P(33.3333%), we use linear interpolation: P(30%) + [P(40%)-P(30%)]/3 = 40% + (43%-40%)/3 = 41%.

Revised exposure rate = [Total in (f)] / [Total in (b)] = 165,041.73/867,557 ≈ **19.02%.**

Note that the exposure rate actually increased, because the decline in net premium was more substantial than the decline in exposure premium (in this case, very modest).

(c) In both cases, with and without the insuring reinsurance, the expected losses for the property per-risk treaty would be (Total Exposure Premium)*(Expected Loss Ratio).

Without inuring surplus-share treaty:
Expected Treaty Losses = (Total Exposure Premium)*(Expected Loss Ratio) = 165,220.13*55% = **$90,871.07**.

With inuring surplus-share treaty:
Expected Treaty Losses = (Total Exposure Premium)*(Expected Loss Ratio) = 165,041.73*55% = **$90,772.95**.

Problem BRP-52. Describe the three general categories of casualty per-occurrence excess-of-loss reinsurance treaties (Clark, pp. 22-23).

Solution BRP-52. The three general categories of casualty per-occurrence excess-of-loss reinsurance treaties are as follows (Clark, p. 22):

1. **Working layer:** The attachment point is low and is expected to be penetrated multiple times per treaty period.
2. **Exposed excess:** The excess layer of reinsurance coverage attaches below the policy limits for some of the policies in the book of business. It is possible for the reinsurer to incur a loss on the treaty if losses on enough underlying policies are close to their limits.
3. **Clash cover:** The attachment point is typically above the limits of any one policy, and the treaty is expected to apply only in cases where multiple policies (including policies of different types) are triggered, or extra-contractual obligations or damages in excess of policy limits are required of the primary insurer.

Problem BRP-53. If a perfect working-layer reinsurance treaty were to be arrived at, what is a paradoxical implication discussed by Clark (p. 23)?

Solution BRP-53. A perfect working-layer reinsurance treaty would produce sufficiently stable results that the ceding company would want to retain the layer.

Problem BRP-54. When gathering the subject premium and historical losses for pricing a casualty per-occurrence excess reinsurance treaty, how should the following aspects be addressed, according to Clark (p. 23)?

(a) Collection of data pertaining to allocated loss-adjustment expenses (ALAE)
(b) Collection of data pertaining to automobile liability and general liability losses
(c) Workers' compensation losses

Solution BRP-54.
(a) ALAE should be captured separately from losses.
(b) For automobile liability and general liability losses, the underlying policy limit should be listed.
(c) Workers' compensation losses should be provided on a full, undiscounted basis.
(Clark, p. 23)

Problem BRP-55. Clark (p. 24) discusses a theoretical problem with regard to capping trended losses at applicable policy limits. Explain the difficulty in determining the right approach to follow on this matter.

Solution BRP-55. It is theoretically desirable to cap trended losses at the limits at which the policy would have been issued, had it been issued during the later time period under consideration. This means that applying the policy's historical limit may not be appropriate, as insurers generally increase policy limits over time in an inflationary environment. If no capping is used, this assumes that policy limits increase at the rate of inflation, which may also not correspond to reality; this approach would also require an adjustment to the subject premium, or else the loss costs will be overstated (Clark, p. 24).

Problem BRP-56. Suppose you are analyzing a casualty per-occurrence excess reinsurance treaty with an attachment point of $500,000 and a limit of $1,000,000.

A particular historical claim has trended loss amount of $900,000 and trended allocated loss-adjustment expenses (ALAE) of $300,000.

(a) What total amount of losses and ALAE would be covered by the treaty if ALAE are allocated *pro rata* with loss?
(b) What total amount of losses and ALAE would be covered by the treaty if ALAE are treated as part of loss?
(c) What is the range of ALAE amounts for this loss, for which treating ALAE as *pro rata* with loss would result in more ALAE being covered by the treaty than treating ALAE as part of loss?

(See Clark, pp. 24-25)

Solution BRP-56.

(a) If ALAE are allocated *pro rata* with loss, then the proportion of ALAE covered is equal to (Loss Covered by Treaty)/(Total Loss) = (900000 – 500000)/(900000) = (400000)/(900000) = 4/9. Therefore, the amount of ALAE covered by the treaty would be (4/9)* $300,000 = $133,333.33, and the total amount covered by the treaty would be 400,000 + 133,333.33 = **$533,333.33**.

(b) If ALAE are treated as part of loss, then the total Loss + ALAE = $1,200,000, and the total amount covered by the treaty would be $1,200,000 - $500,000 = **$700,000.**

(c) If ALAE are treated as part of loss, then the greatest amount of ALAE that the treaty would cover would be the remainder of the reinsurance layer after the loss is covered. The reinsurance layer is for $1,000,000, of which $400,000 would be used up by the loss above the attachment point. Thus, at most $600,000 in ALAE would be covered if ALAE are treated as part of loss. We need to figure out the total amount of ALAE under a *pro rata* allocation, which would lead to more than $600,000 being allocated to the reinsured layer. Since (4/9) of the ALAE is allocated to the reinsured layer under a *pro rata* allocation, this means that total incurred ALAE would need to be greater than ($600,000)/(4/9) = $1,350,000 in order for the *pro rata* allocation to result in a larger covered amount than including the entire ALAE as part of loss.

The desired range is therefore **ALAE > $1,350,000.**

Problem BRP-57. Clark (p. 26) describes four areas of caution to be exercised with regard to using loss-development data from the Reinsurance Association of America (RAA). Describe these areas.

Solution BRP-57. The following cautions should be taken with regard to development data from the Reinsurance Association of America (RAA) (Clark, p. 26):
1. The RAA data are from various companies, and the reporting lag between the event's occurrence and the reinsurer's establishment of a case reserve for it varies by company. Also, the reporting lag for retrocessional business exceeds the reporting lag for ordinary reinsurance.
2. The RAA data are not sufficiently segmented by limits and attachment points, and, even when some segmentation is attempted, the stability of the data is compromised as a result.
3. It is not clear whether the reporting companies consistently adhere to the RAA's standard of excluding asbestos and environmental claims. Also, some long-term exposure claims pertaining to certain products are not excluded, even though they are not relevant to all reinsurance business.
4. It is not clear whether reporting workers' compensation reinsurers consistently treat the tabular discount on large claims.

Problem BRP-58. ***Fill in the blanks*** (Clark, p. 26): Having a very slow development pattern will often produce results showing either _________ or ___________ projected ultimate layer losses by year. The actuary will often need to use ___________ techniques such as the ____________ or ____________ methods to produce a final experience rate.

Solution BRP-58. Having a very slow development pattern will often produce results showing either **zero** or **very high** projected ultimate layer losses by year. The actuary will often need to use **smoothing** techniques such as the **Bornhuetter-Ferguson** or **Cape Cod (Stanard-Bühlmann)** methods to produce a final experience rate.

Problem BRP-59. You are given the following definitions from Clark, p. 27:

x = random variable for size of loss
F(x) = probability a loss is x or smaller, the cumulative distribution function (CDF)
f(x) = density function, first derivative of F(x)
E[x] = expected value or average unlimited loss
E[x;L] = expected value of losses capped at L

In the context of using exposure rating to price a casualty per-occurrence excess reinsurance treaty, provide formulas for the following:

(a) E[x; L] in terms of x, f(x), and L.
(b) $ILF_{L,U}$, the increased-limits factor for a loss capped at U, compared to a loss capped at L.
(c) ELF_L the excess-loss factor for a loss exceeding size L.

Solution BRP-59.
(a) $\mathbf{E[x; L] = {}_0{}^L\!\int(x*f(x)dx) + {}_L{}^\infty\!\int(L*f(x)dx)}$
(b) $\mathbf{ILF_{L,U} = E[x;u]/E[x;L]}$
(c) $\mathbf{ELF_L = (E[x] - E[x;L])/E[x]}$

Problem BRP-60. You are given the following five parameters for a truncated Pareto distribution, as described by Clark, pp. 27-28:

T = Truncation point, "small" losses are below this point, "large" losses follow a Pareto distribution
P = probability of a "small" loss
S = average small-loss severity
B = scale parameter for Pareto distribution
Q = shape parameter for Pareto distribution, $Q \neq 1$

Provide an expression for E[x;L], the expected value of losses capped at L, where L > T.

Solution BRP-60.

$\mathbf{E[x;L] = P*S + [(1-P)/(Q-1)]*[(B + Q*T) - (B+L)*[(B+T)/(B+L)]^Q]}$

Problem BRP-61. You are given the following five parameters for a truncated Pareto distribution, as described by Clark, pp. 27-28:

T = Truncation point, "small" losses are below this point, "large" losses follow a Pareto distribution
P = probability of a "small" loss
S = average small-loss severity
B = scale parameter for Pareto distribution
Q = shape parameter for Pareto distribution, Q ≠ 1

Which of these three factors can be multiplied by the same amount to adjust the scale of the distribution for inflation?

Solution BRP-61. The factors **T, S, and B** can be multiplied by the same amount to adjust the scale of the distribution for inflation.

Problem BRP-62.
(a) For what lines of business is it possible to use a truncated Pareto distribution to model loss severity, according to Clark (p. 27)?
(b) According to Clark (p. 28), what are two limitations of using a truncated Pareto distribution to model loss severity?

Solution BRP-62.
(a) It is possible to use a truncated Pareto distribution to model loss severity for **automobile liability** and **general liability** lines of business.
(b) The two limitations are as follows:
1. The formula for E[x; L] only applies for losses L above the truncation point T. In practice, that parameter would be set well below any treaty attachment point.
2. The excess factors for higher layers become very dependent on the Q (shape) parameter, which must be watched carefully when the curves are updated. (Clark, p. 28)

Problem BRP-63. For a truncated Pareto distribution where the scale parameter B becomes 0 and the shape parameter Q becomes 1, which distribution results? Provide the formula for E[x;L], the expected value of losses capped at L, for this distribution. (Clark, p. 28)

Solution BRP-63. A **loglogistic distribution** results. The formula for E[x;L] is
E[x;L] = P*S + (1-P)*T*[1 – ln(T/L)].

Problem BRP-64. What particular property do expected losses have under a loglogistic distribution? Provide a formula to represent this. (Clark, p. 28)

Solution BRP-64. Under a loglogistic distribution, expected losses in layers are equal if the limit and attachment point are in the same ratio. (Clark, p. 28)

The formula to represent this is as follows:
E[x; U] – E[x; L] = E[x; kU] – E[x; kL] for any constant k.

Problem BRP-65. Clark notes on p. 28 that, under a loglogistic distribution, the relationship E[x; U] – E[x; L] = E[x; kU] – E[x; kL] for any constant k would hold for severities for individual claims, but not necessarily for treaty loss costs. Why might this relationship not hold for treaty loss costs?

Solution BRP-65. This relationship might not hold for treaty loss costs because the loss costs might decrease for higher layers, in which fewer policies are exposed.

Problem BRP-66. Define the term "discrete mixture" (Clark, p. 28).

Solution BRP-66. A discrete mixture is a weighted average of relatively simple curve forms that approximates a more complex but realistic shape. (Clark, p. 28)

Problem BRP-67. If a mixed exponential model is used, where μ_j is the mean of the jth exponential distribution out of N total distributions, the cumulative distribution function $F(x_j) = (1-\exp(-x_j/\mu_j))$, and w_j is the weight attributed to the jth exponential distribution (such that ${}_{j=1}{}^{N}\Sigma(w_j) = 1$), what is the formula for E[x; L], the expected value of loss x, capped at amount L? (Clark, p. 29)

Solution BRP-67. $\mathbf{E[x; L] = {}_{j=1}{}^{N}\Sigma(w_j*\mu_j*(1-\exp(-L/\mu_j)))}$.

Problem BRP-68. Suppose you are given an exponential distribution for size of loss x, with probability density function f(x) = (1/100000)*exp(-x/100000) and cumulative distribution function F(x) = 1 – exp(-x/100000).

(a) What is the unlimited expected value of x, E[x]?
(b) What is the unlimited expected value of x, capped at $300,000, E[x; 300,000]?
(c) What is $ILF_{300000,500000}$, the increased-limit factor for x capped at 500,000, compared to x capped at 300,000?
(d) What is ELF_{300000}, the excess-loss factor above loss size 300,000?

You may use a calculator to perform any integration.

Solution BRP-68.
(a) The mean of an exponential distribution is the parameter θ, where f(x) = (1/θ)*exp(-x/θ). We see that, in the given f(x), θ = **E[x] = 100,000**.
(b) We use the formula $E[x; L] = {}_0{}^{L}\int(x*f(x)d(x)) + {}_L{}^{\infty}\int(L*f(x)d(x))$.
Here, E[x; 300,000] =
${}_0{}^{300,000}\int(x*(1/100000)*\exp(-x/100000)d(x)) + {}_{300,000}{}^{\infty}\int((300000/100000)*\exp(-x/100000)d(x)) =$
80085.17265 + 14936.12051 = **E[x; 300,000] = 95,021.29**.
(c) We use the formula $ILF_{L,U} = E[x; U]/E[x; L]$.
In this case, $ILF_{300000,500000} = E[x; 500,000]/E[x; 300,000]$.
We found in part (b) that E[x; 300,000] = 95021.29.
Now we calculate E[x; 500,000] =
${}_0{}^{500,000}\int(x*(1/100000)*\exp(-x/100000)d(x)) + {}_{500,000}{}^{\infty}\int((500000/100000)*\exp(-x/100000)d(x)) =$

95957.2318 + 3368.9735 = E[x; 500,000] = 99326.2053.
Therefore, E[x; 500,000]/E[x; 300,000] = 99326.2053/95021.29 = **$ILF_{300000,500000}$ = 1.045305**.
(d) We use the formula ELF_L = (E[x]- E[x; L])/E[x]. We know that E[x] = 100,000, and E[x; 300,000] = 95,021.29. Therefore, (100000 - 95021.29)/100000 = **ELF_{300000} = 0.0497871**.

Problem BRP-69. For a certain type of loss, you are given the following five parameters for a truncated Pareto distribution, as described by Clark, pp. 27-28:

T = 15000 = Truncation point, "small" losses are below this point, "large" losses follow a Pareto distribution
P = 0.55 = probability of a "small" loss
S = 10000 = average small-loss severity
B = 19000 =scale parameter for Pareto distribution
Q = 4 = shape parameter for Pareto distribution, Q ≠ 1

Find E[x; 100000], the expected value of the loss, capped at 100,000.

Solution BRP-69.
We use the formula E[x;L] = P*S + [(1-P)/(Q-1)]*[(B + Q*T) – (B+L)*[(B+T)/(B+L)]^Q].

E[x; 100000] = 0.55*10000 + [(1-0.55)/(4-1)]*[(19000 + 4*15000) – (19000 + 100000)*[(19000 + 15000)/(19000 + 100000)]^4] = 5500 + 0.15*(79000 – 119000*0.285714286^4) =
E[x; 100000] = 17231.04956.

Problem BRP-70. For a certain type of loss, you are given the following parameters for a loglogistic distribution, as described by Clark, p. 28:

T = 15000 = Truncation point: "small" losses are below this point.
P = 0.55 = probability of a "small" loss
S = 10000 = average small-loss severity

Find E[x; 100000], the expected value of the loss, capped at 100,000.

Solution BRP-70. We use the formula E[x;L] = P*S + (1-P)*T*[1 – ln(T/L)].
E[x; 100000] = 0.55*10000 + (1-0.55)*15000*(1-ln(15000/100000)) = **E[x; 100000] = 25055.5599.**

Problem BRP-71. For a given loglogistic distribution, you know that E[x; 150,000] = 27792.44938, E[x; 100,000] = 25055.5599, and E[x; 20,000] = 14191.85399. Find E[x; 30,000].

Solution BRP-71. We use the formula E[x; U] – E[x; L] = E[x; kU] – E[x; kL], which applies for any constant k and any values U and L if x follows a loglogistic distribution.
Here, U = 150,000, L = 100,000, and k = (1/5), so kL = 20,000. Thus, kU = 150,000/5 = 30,000.
E[x; kU] = E[x; U] – E[x; L] + E[x; kL].
Here, E[x; 30,000] = E[x; 150,000] - E[x; 100,000] + E[x; 20,000] = 27792.44938 - 25055.5599 + 14191.85399 = **E[x; 30,000] = 16928.74347**.

Problem BRP-72. For a certain type of loss, you are using a mixed exponential model consisting of three exponential distributions, as follows:
• Weight of 0.3 assigned to a distribution with mean 100,000
• Weight of 0.4 assigned to a distribution with mean 200,000
• Weight of 0.3 assigned to a distribution with mean 250,000

Find E[x; 300,000], the expected value of the loss, capped at 300,000.

Solution BRP-72. We use the formula E[x; L] = $_{j=1}{}^{N}\Sigma(w_j*\mu_j*(1\text{-}\exp(\text{-}L/\mu_j)))$.
Here, we are given L = 300000, N = 3, w_1 = 0.3, w_2 = 0.4, w_3 = 0.3, μ_1 = 100000, μ_2 = 200000, and μ_3 = 250000.

Therefore, E[x; 300,000] = 0.3*100000*(1-exp(-300000/100000)) + 0.4*200000*(1-exp(-300000/200000)) + 0.3*250000*(1-exp(-300000/250000)) = **E[x; 300,000] = 143066.4092**.

Problem BRP-73. Let f(x) be the probability density function for the severity distribution for loss amount x, and let E[x; L] be the expected value of the loss, capped at any amount L.

Assume there is a casualty per-occurrence excess reinsurance treaty with PL = Ceding Company Policy Limit, AP = Treaty Attachment Point, and Lim = Treaty Limit. Provide the formula for the exposure factor when exposure rating is used. (Clark, p. 29)

Solution BRP-73.
Exposure Factor = (E[x; min(PL, AP + Lim)] – E[x; min(PL, AP)])/E[x; PL].

Problem BRP-74. For a given loglogistic distribution which a loss type follows, you know that E[x; 150,000] = 27792.44938, E[x; 100,000] = 25055.5599, and E[x; 20,000] = 14191.85399.

The ceding company's policy limit is 150,000.

There is a casualty per-occurrence excess reinsurance treaty whose attachment point is 20,000, and whose limit is 80,000. Calculate the exposure factor for this treaty.

Solution BRP-74. We use the formula Exposure Factor = (E[x; min(PL, AP + Lim)] – E[x; min(PL, AP)])/E[x; PL].

Here, PL = 150000, Lim = 80000, and AP = 20000. Thus,
Exposure Factor = (E[x; min(150000, 20000 + 80000)] – E[x; min(150000, 20000)])/E[x; 150000] = (E[x; 100,000] - E[x; 20,000])/E[x; 150,000] = (25055.5599-14191.85399)/ 27792.44938 = **Exposure Factor = 0.390886955.**

Problem BRP-75. Let f(x) be the probability density function for the severity distribution for loss amount x, and let E[x; L] be the expected value of the loss, capped at any amount L.

Assume there is a casualty per-occurrence excess reinsurance treaty with PL = Ceding Company Policy Limit, applied to losses only.

Also assume that the treaty includes allocated loss-adjustment expenses (ALAE) in proportion to losses. Let e denote such ALAE as a percent of loss, capped at PL.

Let AP = Treaty Attachment Point, applying to ALAE plus loss capped at PL, and Lim = Treaty Limit, applying to ALAE plus loss capped at PL.

(a) Provide the formula for the exposure factor when exposure rating is used.
(b) What is the key assumption of this approach regarding ALAE?
(c) Why, according to Clark, is this not an accurate assumption? (Clark, p. 29)

Solution BRP-75.
(a) Exposure Factor =
(E[x; min(PL, [AP + Lim]/[1+e])] – E[x; min(PL, AP/[1+e])])/E[x; PL].
(b) The key assumption is that ALAE varies directly with capped indemnity loss.
(c) This is not an accurate assumption, because ALAE is not a constant percent of any given loss. Some losses close without indemnity payments but may still incur large ALAE. Clark observes that as the size of a loss increases, the ALAE as a percent of loss will tend to decrease. Thus, the assumption that ALAE is directly proportional to loss would overstate the expected amounts in higher layers. (Clark, p. 29)

Problem BRP-76. ***Fill in the blanks*** (Clark, p. 30): A limitation of the exposure-rating approach that treats ALAE as proportional to loss is that an exposure factor of __________ will be applied to high layers which are indeed exposed. A more refined analysis of the effect of ALAE would require modeling of ___________________________.

Solution BRP-76. A limitation of the exposure-rating approach that treats ALAE as proportional to loss is that an exposure factor of **zero** will be applied to high layers which are indeed exposed. A more refined analysis of the effect of ALAE would require modeling of **how ALAE varies with loss size**.

Problem BRP-77.
(a) What are two possible types of proportional treaties on excess business? Briefly describe each type.
(b) Which of these types of treaties will typically require the ceding company to use increased-limits factors to price the portion of its policies exposing the treaty? (Clark, p. 30)

Solution BRP-77.
(a) Two possible types of proportional treaties on excess business are as follows:
1. Quota-share basis: Reinsurer takes a set percent of each contract the ceding company writes.
2. "Cessions" basis: Percent ceded depends on the attachment point and limit on each policy.
(b) The **cessions-basis treaty** will typically require the ceding company to use increased-limits factors to price the portion of its policies exposing the treaty. (Clark, p. 30)

Problem BRP-78. According to Clark (p. 30), what is the NCCI approximation for the excess-loss factor $ELF_L = (E[x]- E[x; L])/E[x]$. Provide an approximation formula and explain it conceptually.

Solution BRP-78. The approximation formula is $\mathbf{ELF_L = a*L^{-b}}$, an inverse power curve, where a and b are parameters.

Problem BRP-79.
(a) Unlike policies for other lines of business, what feature do workers' compensation policies *not* have?
(b) What aspect of workers' compensation policies is present instead of this feature?
(c) How does this difference affect the calculation of the exposure factor for excess reinsurance of workers' compensation business? Provide a formula. (Clark, p. 30)

Solution BRP-79.
(a) Workers' compensation policies **do not have policy limits**.
(b) Instead, workers' compensation policies are subject to limitations on annual benefits, as provided by state law.
(c) The exposure factor for excess reinsurance of workers' compensation business is calculated only using the treaty attachment point (AP) and treaty limit (Lim):
Exposure Factor = $\mathbf{ELF_{AP} - ELF_{AP+Lim}}$. (Clark, p. 30)

Problem BRP-80. You are evaluating a casualty excess per-occurrence reinsurance treaty for underlying workers' compensation business. The treaty has an attachment point of 500,000 and a limit of 1,000,000.

Exposure rating is being used to price this reinsurance treaty. To calculate the excess-loss factors, an inverse power curve of the form $a*L^{-b}$ is used, where a = 0.5 and b = -0.2.

(a) Calculate the excess-loss factor $ELF_{500,000}$ for the treaty's attachment point.
(b) Calculate the exposure factor for this reinsurance treaty.
(c) If the standard workers' compensation premium for the underlying business is 8,000,000, and the expected loss ratio is 66%, calculate the expected treaty losses.
(d) Calculate the loss cost for the treaty.

Solution BRP-80.
(a) We use the formula $ELF_L = a*L^{-b}$. Here, $\mathbf{ELF_{500,000}}$ = $0.5*500000^{-0.2}$ = **0.0362389832**.
(b) We use the formula Exposure Factor = $ELF_{AP} - ELF_{AP+Limit}$. We have already calculated $ELF_{AP} = ELF_{500,000}$ = 0.0362389832. Now we calculate $ELF_{AP+Limit}$ = $ELF_{1,500,000} = 0.5*1500000^{-0.2}$ = 0.029090538.
Thus, **Exposure Factor =** 0.0362389832 - 0.029090538 = **0.0071484452**.
(c) The expected treaty losses are equal to (Standard Premium)*(Expected Loss Ratio)*(Exposure Factor) = 8,000,000*0.66*0.0071484452 = **Expected Treaty Loss = 37743.79066**.
(d) The loss cost is equal to (Expected Treaty Loss)/(Standard Premium) = 37743.79066/8000000 = 0.004717974 = **0.4717974%**.

Problem BRP-81.
(a) Briefly describe an umbrella policy.
(b) If a ceding company's business covered by a casualty per-occurrence excess reinsurance treaty includes umbrella policies, what is the best way to consider the combination of primary and umbrella business, according to Clark (p. 31)?
(c) What situation would complicate this approach?

Solution BRP-81.
(a) An umbrella policy provides coverage in excess of an underlying retention and "drops down" if an underlying aggregate limit is exhausted.
(b) The best way to consider the combination of the primary and umbrella business is to consider it as a single policy with a higher limit.
(c) If the umbrella policies are above primary policies for *other carriers,* the approach in part (b) is complicated by this.

Problem BRP-82. ***Fill in the blanks*** (Clark, p. 32): When applying *experience rating* to pricing a casualty per-occurrence excess reinsurance treaty that includes coverage for umbrella policies, the main difficulty is in ______________. The limit on the underlying policy should be added to losses before ______________ and subtracted afterward.

Solution BRP-82. When applying experience rating to pricing a casualty per-occurrence excess reinsurance treaty that includes coverage for umbrella policies, the main difficulty is in **selecting the appropriate trend factor**. The limit on the underlying policy should be added to losses before **the application of trend** and subtracted afterward.

Problem BRP-83.
(a) Provide the formula for trended loss when applying *experience rating* to pricing a casualty per-occurrence excess reinsurance treaty that includes coverage for umbrella policies. In the formula, let T = trended loss, L = untrended loss, F = trend factor, and U = underlying limit.
(b) What losses will the procedure represented by this formula leave out? (Clark, p. 32)

Solution BRP-83.
(a) $\mathbf{T = (L+U)*F - U}$.
(b) The procedure will leave out losses from the underlying policy which historically did not exhaust the underlying limit, but which would have exhausted it after the application of a trend factor. (Clark, p. 32)

Problem BRP-84. When applying *exposure rating* to pricing a casualty per-occurrence excess reinsurance treaty that includes coverage for umbrella policies, you are given the following terms:

UL = Limit of underlying policies (attachment point of umbrella)
PL = Policy limit of umbrella
AP = Treaty attachment point
Lim = Treaty limit
E[x; L] = expected value of loss x, capped at any amount L

(a) Provide a formula for the exposure factor on the excess policy. (Clark, p. 32)
(b) What possibility does this formula leave out? (Clark, p. 33)

Solution BRP-84.
(a) Exposure Factor = (E[x; min(UL + PL, UL + AP + Lim)] – E[x; min(UL + PL, UL + AP)])/(E[x; UL + PL] – E[x; UL]).
(b) This formula leaves out the possibility of the **"drop down" feature of the umbrella policy**. (Clark, p. 33)

Problem BRP-85. You are applying exposure rating to price a casualty per-occurrence excess reinsurance treaty that includes coverage for umbrella policies. The following information is given:

• The attachment point of the umbrella policies is $500,000.
• The umbrella policies have limits of $1,200,000.
• The treaty attachment point is $400,000.
• The treaty limit is $800,000.

You are also given the following limited expected values of loss amount x:
E[x; 400,000] = 222,222
E[x; 500,000] = 333,333
E[x; 800,000] = 444,444
E[x; 900,000] = 488,888
E[x; 1,200,000] = 555,555
E[x; 1,300,000] = 566,666
E[x; 1,600,000] = 622,222
E[x; 1,700,000] = 644,444
E[x; 2,000,000] = 700,000

Calculate the exposure factor for this reinsurance treaty.

Solution BRP-85. We use the formula
Exposure Factor = (E[x; min(UL + PL, UL + AP + Lim)] – E[x; min(UL + PL, UL + AP)])/(E[x; UL + PL] – E[x; UL]).

Here, UL = 500,000, PL = 1,200,000, AP = 400,000, and Lim = 800,000.

Thus, Exposure Factor = (E[x; min(500,000 + 1,200,000, 500,000 + 400,000 + 800,000)] – E[x; min(500,000 + 1,200,000, 500,000 + 400,000)])/(E[x; 500,000 + 1,200,000] – E[x; 500,000]) =

(E[x; 1,700,000] – E[x; 900,000])/(E[x; 1,700,000] – E[x; 500,000]) =
(644,444 - 488,888)/(644,444 - 333,333) = **Exposure Factor = 0.500001607**.

Problem BRP-86. ***Fill in the blanks*** (Clark, p. 33): For working layer excess reinsurance, the ceding company is often willing to retain _________ [more or less?] of the losses. In these cases a(n) __________ may be used. The treaty then becomes a(n) ___________ of ____________ cover, where the ______________ are the per-occurrence excess losses in the layer.

Solution BRP-86. For working layer excess reinsurance, the ceding company is often willing to retain **more** of the losses. In these cases an **annual aggregate deductible (AAD)** may be used. The treaty then becomes an **excess** of **aggregate** cover, where the **aggregate losses** are the per-occurrence excess losses in the layer. (Clark, p. 33)

Problem BRP-87. You are using an aggregate distribution model to estimate the excess charge factor φ_{AAD} for a given annual aggregate deductible (AAD). Let g(y) be the distribution of aggregate losses in the reinsurance layer, and let E[y] be the expected value of the loss. Provide the formula for the excess charge factor φ_{AAD}. (Clark, p. 34)

Solution BRP-87. $\boldsymbol{\varphi_{AAD} = {}_{AAD}{}^{\infty}\int[(y - AAD)*g(y)dy]/E[y]}$**.**

Problem BRP-88. You are using an aggregate distribution model to estimate the excess charge factor φ_{AAD} for a given annual aggregate deductible (AAD) of 300,000 that applies to a casualty per-occurrence excess reinsurance treaty. You know that aggregate losses y in the reinsured layer follow an exponential distribution with mean 100,000 and probability density function g(y) = (1/100000)*exp(-y/100000).
(a) Calculate $\varphi_{300,000}$.
(b) If the loss cost for the layer gross of the AAD is 15%, what is the net loss cost after application of the AAD?

You may use a calculator to perform any integration.

Solution BRP-88.
(a) We use the formula $\varphi_{AAD} = {}_{AAD}{}^{\infty}\int[(y - AAD)*g(y)dy]/E[y]$.
We know that E[y], the mean of the given exponential distribution, is 100,000.
Thus, $\varphi_{300,000} = {}_{300,000}{}^{\infty}\int[(y - 300,000)*(1/100000)*\exp(-y/100000)dy]/100,000$ =
$\boldsymbol{\varphi_{300,000}}$ **= 0.0497870684**.
(b) The net loss cost is equal to the loss cost gross of the AAD, multiplied by φ_{AAD}. Here, the net loss cost is 15%*0.0497870684 = **0.7468060255%.**

Problem BRP-89. ***Fill in the blanks*** (Clark, p. 35): Experience rating for workers' compensation may be distorted depending on how _________ are taken into account. A way to avoid this distortion is to collect _________________ to project _______________.

Solution BRP-89. Experience rating for workers' compensation may be distorted depending on how **tabular discounts** are taken into account. A way to avoid this distortion is to collect **sufficient information for individual claimants** to project **their expected costs into the treaty layer**. (Clark, p. 35)

Problem BRP-90. ***Fill in the blanks*** (Clark, p. 36): As a general rule, aggregate distribution models produce results which are very sensitive to the ____________. Whenever possible, ______ analysis on the ________, or even several approaches, should be used.

Solution BRP-90. As a general rule, aggregate distribution models produce results which are very sensitive to the **input assumptions**. Whenever possible, **sensitivity** analysis on the **parameters**, or even several approaches, should be used. (Clark, p. 36)

Problem BRP-91. If an actuary has five or more years of loss ratios on a surplus-share reinsurance treaty, what approach could be used to price a sliding-scale commission using an empirical distribution? (Clark, p. 36)

Solution BRP-91. The commission can be calculated as if current treaty terms had been in effect over the historical period (adjusted to current rate level). (Clark, p. 36)

Problem BRP-92. What are three caveats mentioned by Clark (p. 36) to the use of an empirical distribution, based on historical experience, in pricing a reinsurance treaty?

Solution BRP-92. The following are the three caveats mentioned by Clark (p. 36):
1. The experience does not take into account all possible outcomes, and may miss the possibility of events outside of what has been observed.
2. If the volume or mix of business has been changing, then the volatility of the future period may be very different than the historical period.
3. If loss development has been performed using a Bornhuetter-Ferguson or Cape Cod method, then the historical periods may present an artificially smooth sequence of loss ratios that does not reflect future volatility.

Problem BRP-93. Let $\Phi(x)$ be the value of the Standard Normal distribution for input x. Let G(y) be the cumulative distribution function of a lognormal distribution with parameters μ and σ.

(a) Provide the formula for the cumulative distribution function G(y).
(b) Let E[y] be the mean of the lognormal distribution and CV be the coefficient of variation. Express σ^2 as a function of any of these terms.
(c) Let E[y] be the mean of the lognormal distribution and CV be the coefficient of variation. Express μ as a function of any of these terms.
(d) Let E[y; L] be the expected loss function for loss amount y, limited at L. Provide the formula for E[y; L].
(e) Let φ_L be the *excess charge function* with respect to losses above amount L. Provide the formula for φ_L using limited and unlimited expected values of y.
(f) If loss amount y can only be within the range L < y < U, where L is the lower end of the range and U is the upper end of the range, what is the formula for E[y | L < y < U], the conditional expected value of y, given these constraints?

(Clark, p. 37)

Solution BRP-93.
(a) $G(y) = \Phi([\ln(y) - \mu]/\sigma)$.
(b) $\sigma^2 = \ln(CV^2 + 1)$
(c) $\mu = \ln(E[y]) - \sigma^2/2$
(d) $E[y; L] = \exp(\mu + \sigma^2/2)*\Phi([\ln(L)-\mu-\sigma^2]/\sigma) + L*[1-\Phi([\ln(L) - \mu]/\sigma)]$
(e) $\varphi_L = (E[y] - E[y; L])/E[y]$
(f) $E[y \mid L < y < U] = \exp(\mu + \sigma^2/2)*[\Phi([\ln(U)-\mu-\sigma^2]/\sigma) - \Phi([\ln(L)-\mu-\sigma^2]/\sigma)]/[\Phi([\ln(U) - \mu]/\sigma) - \Phi([\ln(L) - \mu]/\sigma)]$

Problem BRP-94. Suppose that loss amount y follows a lognormal distribution with parameters $\mu = 9.512925465$ and $\sigma^2 = 4$.

(a) What is the mean (E[y]) of this lognormal distribution?
(b) What is the coefficient of variation (CV) of this lognormal distribution?
(c) What is the standard deviation (SD) of this lognormal distribution?
(d) What is G(200,000), the cumulative distribution function at loss amount 200,000?
(e) What is E[y; 200,000], the expected value of the loss, limited at 200,000?
(f) What is $\varphi_{200,000}$, the excess charge factor for losses above 200,000?
(g) If losses were limited to being between 50,000 and 300,000, what would be the conditional expected value $E[y \mid 50,000 < y < 300,000]$?

You may use Excel or a calculator to evaluate values of the Standard Normal distribution.

Solution BRP-94.
(a) We use the formula $\mu = \ln(E[y]) - \sigma^2/2$. Thus, $\ln(E[y]) = \mu + \sigma^2/2$, and $E[y] = \exp(\mu + \sigma^2/2)$. Here, $\mu = 9.512925465$ and $\sigma^2 = 4$. Thus, E[y] = exp(9.512925465 + 4/2) = **E[y] = 100,000**.

(b) We use the formula $\sigma^2 = \ln(CV^2 + 1)$. Thus, $\exp(\sigma^2) = CV^2 + 1 \rightarrow CV^2 = \exp(\sigma^2) - 1 \rightarrow CV = \sqrt{(\exp(\sigma^2) - 1)} = \sqrt{(\exp(4) - 1)}$ = **CV = 7.321075743**.

(c) Because CV = SD/E[y], it follows that SD = CV*E[y] = 100,000*7.321075743 = **SD = 732,107.5743**.

(d) We use the formula $G(y) = \Phi([\ln(y) - \mu]/\sigma)$. Here, $\mu = 9.512925465$ and $\sigma = 2$.
Thus, G(200,000) = Φ([ln(200,000) – 9.512925465]/2) = Φ(1.34657359), which can be solved in Excel using the input "=NORMSDIST(1.34657359)". The answer is **G(200,000) = 0.910941**.

(e) We use the formula $E[y; L] = \exp(\mu + \sigma^2/2)*\Phi([\ln(L)-\mu-\sigma^2]/\sigma) + L*[1-\Phi([\ln(L) - \mu]/\sigma)]$.
This can be reduced to $E[y; L] = E[y]*\Phi([\ln(L)-\mu-\sigma^2]/\sigma) + L*[1-G(L)]$.
We have already calculated E[y] = 100000 and G(200000) = 0.910941.
Thus, E[y; 200,000] = 100000* Φ([ln(200000)- 9.512925465-4]/2) + 200000*(1-0.910941) = 100000* Φ(-0.65342641) + 17811.8.
The Excel input for this is "=100000*NORMSDIST(-0.65342641) + 17811.8". The answer is **E[y; 200,000] = 43,485.87087**.

(f) We use the formula $\varphi_L = (E[y] - E[y; L])/E[y]$. Thus, $\varphi_{200,000} = (E[y] - E[y; 200,000])/E[y] = (100000 - 43485.87087)/100000 =$ $\boldsymbol{\varphi_{200,000} = 0.565141291}$.

(g) We use the formula $E[y \mid L < y < U] = \exp(\mu + \sigma^2/2)*[\Phi([\ln(U)-\mu-\sigma^2]/\sigma) - \Phi([\ln(L)-\mu-\sigma^2]/\sigma)]/[\Phi([\ln(U) - \mu]/\sigma) - \Phi([\ln(L) - \mu]/\sigma)]$. Here, L = 50000 and U = 300000. The expression $\exp(\mu + \sigma^2/2)$ is the same as E[y].
Thus, $E[y \mid 50,000 < y < 300,000] = E[y]*[\Phi([\ln(300000)-\mu-\sigma^2]/\sigma) - \Phi([\ln(50000)-\mu-\sigma^2]/\sigma)]/[\Phi([\ln(300000) - \mu]/\sigma) - \Phi([\ln(50000) - \mu]/\sigma)] =$
$100000*[\Phi([\ln(300000)- 9.512925465-4]/2) - \Phi([\ln(50000)- 9.512925465-4]/2)]/[\Phi([\ln(300000) - 9.512925465]/2) - \Phi([\ln(50000) - 9.512925465]/2)] =$
$100000*[\Phi(-0.450693856) - \Phi(-1.34657359)]/[\Phi(1.549306144)-\Phi(0.65342641)]$.
The Excel input for this is "=100000*(NORMSDIST(-0.450693856)-NORMSDIST(-1.34657359))/(NORMSDIST(1.549306144)-NORMSDIST(0.65342641))". **The answer is $\boldsymbol{E[y \mid 50,000 < y < 300,000] = 120888.5557}$.**

Problem BRP-95. Why, according to Clark (p. 38), is the formula for the expected value in a given range useful for pricing reinsurance agreements with adjustable features?

Solution BRP-95. Most adjustable features can be broken down into piecewise linear functions, and only the expected value is needed within each linear range. (Clark, p. 38)

Problem BRP-96. Suppose that loss amount y follows a lognormal distribution with parameters $\mu = 9.512925465$ and $\sigma^2 = 4$.

Pricing for a reinsurance treaty for these losses includes a swing-rated adjustment, where the minimum premium is equal to 5% of the subject premium. The reinsurance premium increases at a rate of 40% of actual losses during the treaty period, up to a maximum of 15% of the subject premium.

The subject premium for the ceding insurer is 350,000.

Find the expected reinsurance premium for this treaty.

You may use Excel or a calculator to evaluate values of the Standard Normal distribution.

Solution BRP-96. We will use the general formula for the conditional expected value of a lognormal distribution: $E[y \mid L < y < U] = \exp(\mu + \sigma^2/2)*[\Phi([\ln(U)-\mu-\sigma^2]/\sigma) - \Phi([\ln(L)-\mu-\sigma^2]/\sigma)]/[\Phi([\ln(U) - \mu]/\sigma) - \Phi([\ln(L) - \mu]/\sigma)] = E[y]*[\Phi([\ln(U)-\mu-\sigma^2]/\sigma) - \Phi([\ln(L)-\mu-\sigma^2]/\sigma)]/[\Phi([\ln(U) - \mu]/\sigma) - \Phi([\ln(L) - \mu]/\sigma)]$

There are three ranges for y that we will need to consider, based on the loss amounts that would be subject to the swing adjustment and the loss amounts that would not be.

The minimum premium of 5% of the subject premium (0.05*350000 = 17500) would not be exceeded by the swing-rated adjustment unless losses at least were 5%/40% = 12.5% of subject premium = 0.125*350000 = 43750.

The maximum premium of 15% of the subject premium (0.15*350000 = 52500) would be reached when losses reach 15%/40% = 37.5% of subject premium = 0.375*350000 = 131250.

Thus, we can state the following:
• For losses below 43750, the reinsurance premium will be 17500.
• For losses between 43750 and 131250, the reinsurance premium will be 0.4*E[y │ 43750 < y < 131250].
• For losses above 131250, the reinsurance premium will be 52500.

Let G(y) be the cumulative distribution function for the given lognormal distribution.

The expected reinsurance premium would be equal to G(43750)*17500 + 0.4*E[y │ 43750 < y < 131250]*[G(131250) – G(43750)] + [1-G(131250)]*52500.

We use the formula G(y) = Φ([ln(y) – μ]/σ) to calculate G(43750) = Φ([ln(43750) – 9.512925465]/2) = Φ(0.586660713), which can be solved in Excel using the input "=NORMSDIST(0.586660713)" – providing an answer of G(43750) = 0.721284201.

Likewise, we calculate G(131250) = Φ([ln(131250) – 9.512925465]/2) = Φ(1.135966858), which can be solved in Excel using the input "=NORMSDIST(1.135966858)" – providing an answer of G(131250) = 0.872014782.

Recall that the unconditional expected value of this lognormal distribution is exp(9.512925465 + 4/2) = E[y] = 100,000.
Next, we begin calculate E[y │ 43750 < y < 131250] = E[y]*[Φ([ln(131250)-μ-σ^2]/σ) - Φ([ln(43750)-μ-σ^2]/σ)]/[Φ([ln(131250) – μ]/σ) - Φ([ln(43750) – μ]/σ)] = 100000*[Φ([ln(131250)-μ-σ^2]/σ) - Φ([ln(43750)-μ-σ^2]/σ)]/[G(131250) – G(43750)].
Note that the denominator in this term can cancel out the multiplication by [G(131250) – G(43750)] in our desired calculation for the expected reinsurance premium. At this point, it is easiest to focus on directly determining
0.4*E[y │ 43750 < y < 131250]*[G(131250) – G(43750)] = 0.4*100000*[Φ([ln(131250)-μ-σ^2]/σ) - Φ([ln(43750)-μ-σ^2]/σ)] = 40000*[Φ([ln(131250)- 9.512925465-4]/2) - Φ([ln(43750)-9.512925465-4]/2)] = 40000*[Φ(-0.864033142) - Φ(-1.413339287)], for which the Excel input is "=40000*(NORMSDIST(-0.864033142) - NORMSDIST(-1.413339287))". The result is 4600.273938.

Now we can complete the calculation for expected reinsurance premium:
0.721284201*17500 + 4600.273938 + (1-0.872014782)*52500 = **Expected reinsurance premium = 23941.9714**.

Problem BRP-97. What is an advantage of using a single continuous distribution to model reinsurance losses? (Clark, p. 38)

Solution BRP-97. The single continuous distribution model is relatively simple to use, even when source data are limited. The model can provide a reasonable fit even when frequency and severity distributions are not known. (Clark, p. 38)

Problem BRP-98. (a) What are two shortcomings of using a single continuous distribution function to model reinsurance losses? (See Clark, p. 38.)
(b) Briefly explain how a collective risk model differs from using a single continuous distribution function. (See Clark, pp. 36.)

Solution BRP-98. (a) Two shortcomings of using a single continuous distribution function to model reinsurance losses are that (1) the distribution does not permit a scenario where the loss amount is zero (since x = 0 is the smallest independent variable, the cumulative distribution function is zero at x= 0, implying a zero probability) and (2) the impact on the distribution of changing the treaty limits (e.g., the per-occurrence limits) is not easy to determine.
(b) A collective risk model employs a probability distribution to model the severity of each loss, and the number of losses also follows its own probability distribution. This enables explicit recognition of both frequency and severity of losses.

Problem BRP-99. ***Fill in the blanks*** (Clark, p. 39): The recursive formula is a convenient tool for calculating an aggregate distribution for scenarios of ______ frequency. The frequency distribution is assumed to be __________, ___________, or __________, and the severity distribution is defined in _______________.

Solution BRP-99. The recursive formula is a convenient tool for calculating an aggregate distribution for scenarios of **low** frequency. The frequency distribution is assumed to be **Poisson**, **negative binomial**, or **binomial**, and the severity distribution is defined in **discrete steps**.

Problem BRP-100. You are given a Poisson distribution with Pr(n) = λ^n*exp(-λ)/(n!). What is the recursive form of this distribution? Specify Pr(0), and then specify Pr(n) in terms of Pr(n-1), where n is an integer > 0. (Clark, p. 39)

Solution BRP-100.
Pr(0) = exp(-λ)
Pr(n) = (λ/n)*Pr(n-1)

Problem BRP-101. ***Fill in the blanks*** (Clark, p. 39): When using recursion formula, each possible severity must be __________ from the preceding amount. The largest severity may be set equal to the ______________ on an excess treaty, or to the _________ times a loading for _________.

Solution BRP-101. When using recursion formula, each possible severity must be **equally spaced** from the preceding amount. The largest severity may be set equal to the **per-occurrence limit** on an excess treaty, or to the **limit** times a loading for **allocated loss-adjustment expenses (ALAE)**.

Problem BRP-102. Losses for a particular ceding insurer are modeled using a Poisson distribution with parameter $\lambda = 3$ for frequency and the following discrete distribution for severity:

Notation	Severity	Probability
S_1	100,000	0.33
S_2	200,000	0.15
S_3	300,000	0.39
S_4	400,000	0.13

(a) What is the probability of zero losses?
(b) What is the probability that the total loss amount will be 100,000?
(c) What is the probability that the total loss amount will be 200,000?
(d) What is the probability that the total loss amount will be 300,000?

Solution BRP-102.
(a) The probability of zero losses is Pr(0) = exp(-λ) = exp(-3) = **Pr(0) = 0.0497870684**.
(b) The only way in which the total loss amount will be 100,000 is if there is one loss with amount 100,000. We want to therefore find Pr(1)* (S_1). We know that (S_1) = 0.33.
For Pr(1), we use the recursive formula Pr(n) = (λ/n)*Pr(n-1). Thus, Pr(1) = (λ/1)*Pr(0) = 3*exp(-3) = 0.1493612051. Thus, Pr(Total Loss Amount = 100,000) = 0.1493612051*0.33 = **0.0492891977.**
(c) There are two ways in which the total loss amount could be 200,000: 2 losses of 100,000 each, or 1 loss of 200,000. We want to therefore find Pr(1)*(S_2) + Pr(2)*(S_1)*(S_1).
We know that Pr(1) = 0.1493612051, and so Pr(2) = (λ/2)*Pr(1) = (3/2)*0.1493612051 = 0.2240418077. Thus, Pr(Total Loss Amount = 200,000) = 0.1493612051*0.15 + 0.2240418077*0.33*0.33 = **0.046802334**.
(d) The following are the ways in which the total loss amount could be 300,000:
• 3 losses of amount 100,000 each.
• 2 losses, one of amount 200,000, the other of amount 100,000. (Note that these could occur in either order, so we have to include two terms to account for this fact.)
• 1 loss of amount 300,000.
We want to therefore find Pr(1)*(S_3) + Pr(2)*(S_1)*(S_2) + Pr(2)*(S_2)*(S_1) + Pr(3)*(S_1)*(S_1)*(S_1) = Pr(1)*(S_3) + 2*Pr(2)*(S_1)*(S_2) + Pr(3)*$(S_1)^3$.
We know that Pr(2) = 0.2240418077, so Pr(3) = (λ/3)*Pr(2) = (3/3)*0.2240418077 = 0.2240418077.
Thus, Pr(Total Loss Amount = 300,000) = 0.1493612051*0.39 + 2*0.2240418077*0.33*0.15 + 0.2240418077*0.33^3 = Pr(Total Loss Amount = 300,000) = **0.088482399**.

Problem BRP-103. You are given a Poisson frequency distribution with Pr(n) = λ^n*exp(-λ)/(n!). There is also a discrete severity distribution with S_i denoting the probability of the ith loss amount, where the loss amounts are evenly spaced. Let A_k be the probability of the kth aggregate loss amount. What is the recursive formula for A_k in terms of the probabilities for lower aggregate loss amounts, A_{k-i}, for integers i, such that $1 \le i < k$? (Clark, p. 40)

Solution BRP-103. $\mathbf{A_k = {}_{i=1}{}^{k}\Sigma[(\lambda/k)*i*S_i*A_{k-i}]}$

Problem BRP-104. Losses for a particular ceding insurer are modeled using a Poisson distribution with parameter $\lambda = 3$ for frequency and the following discrete distribution for severity:

Notation	Severity	Probability
S_1	100,000	0.33
S_2	200,000	0.15
S_3	300,000	0.39
S_4	400,000	0.13

Fill in the remainder of the following table (for which the already-filled entries were based on the work in Solution BRP-102). (See Clark, p. 40, for an example of the method to be used.)

Amount	Identifier (i)	Severity Probability (S_i)	Aggregate Probability (A_i)
0	0	0	0.0497870684
100,000	1	0.33	0.0492891977
200,000	2	0.15	0.046802334
300,000	3	0.39	0.088482399
400,000	4	0.13	
500,000	5	0	
600,000	6	0	

Solution BRP-104. We use the formula $A_k = {}_{i=1}{}^{k}\Sigma[(\lambda/k)*i*S_i*A_{k-i}]$ for k = 4, 5, and 6.

First we find $A_4 = (\lambda/4)*(1*S_1*A_3 + 2*S_2*A_2 + 3*S_3*A_1 + 4*S_4*A_0) =$ $(3/4)*(1*0.33*0.088482399 + 2*0.15*0.046802334 + 3*0.39*0.0492891977 +$ $4*0.13*0.0497870684) =$ $\mathbf{A_4 = 0.095098147}$.

Then we find $A_5 = (\lambda/5)*(1*S_1*A_4 + 2*S_2*A_3 + 3*S_3*A_2 + 4*S_4*A_1) =$ $(3/5)*(1*0.33*0.095098147 + 2*0.15*0.088482399 + 3*0.39*0.046802334 +$ $4*0.13*0.0492891977) =$ $\mathbf{A_5 = 0.082989733}$.

We conclude by finding $A_6 = (\lambda/6)*(1*S_1*A_5 + 2*S_2*A_4 + 3*S_3*A_3 + 4*S_4*A_2) =$ $(3/6)*(1*0.33*0.082989733 + 2*0.15*0.095098147 + 3*0.39*0.088482399 +$ $4*0.13*0.046802334) =$ $\mathbf{A_6 = 0.091888838}$.

Thus, the filled-out table will be as follows:

Amount	Identifier (i)	Severity Probability (S_i)	Aggregate Probability (A_i)
0	0	0	0.0497870684
100,000	1	0.33	0.0492891977
200,000	2	0.15	0.046802334
300,000	3	0.39	0.088482399
400,000	4	0.13	**0.095098147**
500,000	5	0	**0.082989733**
600,000	6	0	**0.091888838**

Problem BRP-105. You are given a Poisson frequency distribution with $Pr(n) = \lambda^n*\exp(-\lambda)/(n!)$. There is also a discrete severity distribution with S_i denoting the probability of the ith loss amount, where the loss amounts are evenly spaced. Let L_i be the ith possible single loss amount, and let k be the total possible number of single loss amounts.

(a) What is the formula for the mean of the *aggregate* loss distribution (which combines frequency and severity)?
(b) What is the formula for the variance of the *aggregate* loss distribution? (Clark, p. 40)

Solution BRP-105.
(a) Mean = $\lambda* {}_{i=1}{}^{k}\Sigma[L_i*S_i]$
(b) Variance = $\lambda* {}_{i=1}{}^{k}\Sigma[L_i{}^2*S_i]$

Problem BRP-106. Losses for a particular ceding insurer are modeled using a Poisson distribution with parameter $\lambda = 3$ for frequency and the following discrete distribution for severity:

Notation	Severity	Probability
S_1	100,000	0.33
S_2	200,000	0.15
S_3	300,000	0.39
S_4	400,000	0.13

(a) What is the mean of the *aggregate* loss distribution?
(b) What is the standard deviation of the *aggregate* loss distribution?

Solution BRP-106.
(a) We use the formula Mean = $\lambda* {}_{i=1}{}^{k}\Sigma[L_i*S_i]$ = 3*(100000*0.33 + 200000*0.15 + 300000*0.39 + 400000*0.13) = **Mean = 696,000**.
(b) We use the formula Variance = $\lambda* {}_{i=1}{}^{k}\Sigma[L_i{}^2*S_i]$. The standard deviation is the square root of the variance. Thus, Standard Deviation = $\sqrt{}(3*(100000^2*0.33 + 200000^2*0.15 + 300000^2*0.39 + 400000^2*0.13))$ = **Standard Deviation = 442,266.8877**.

Problem BRP-107. Clark (p. 41) provides a general recursive formula for aggregate-loss probabilities where the aggregate distribution involves a frequency distribution that utilizes parameters a and b, defined differently for each frequency distribution. The general recursive formula is $A_k = {}_{i=1}{}^{k}\Sigma[(a + b*i/k)*S_i*A_{k-i}]$.
(a) Define a and b for a Poisson distribution, with $Pr(n) = \lambda^n*\exp(-\lambda)/(n!)$.
(b) Define a and b for a negative binomial distribution, with $Pr(n) = C(\alpha+n-1, n)*p^\alpha*(1-p)^n$.
(c) Define a and b for a binomial distribution, with $Pr(n) = C(M, n)*p^n*(1-p)^{M-n}$.

Solution BRP-107.
(a) Poisson distribution: a = 0, b = λ
(b) Negative binomial distribution: a = (1-p), b = $(\alpha-1)*(1-p)$
(c) Binomial distribution: a = p/(p-1), b = (M+1)*p/(1-p)

Problem BRP-108.
(a) According to Clark (p. 41), what are two main advantages of using a recursive formula?
(b) According to Clark (p. 41), what are two main disadvantages of using a recursive formula?

Solution BRP-108.
(a) Advantages of using a recursive formula (any 2 below will suffice):
1. Recursive formulas are simple to work with.
2. Recursive formulas provide an accurate handling of low-frequency scenarios.
3. The number of points evaluated on the severity distribution can be expanded to closely approximate continuous curves. (Clark, p. 41)

(b) Disadvantages of using a recursive formula:
1. For higher expected frequencies, the calculation is inconvenient, because all the probabilities up to the desired level must be calculated.
2. Only a single severity distribution can be used in the analysis. (Clark, p. 41)

Problem BRP-109.
(a) Provide a general definition of a collective risk model.
(b) In what two situations would a collective risk model that is more advanced than a recursive method need to be used?
(c) How are most aggregate loss models broader than the recursive model in terms of their treatment of severity distributions? (Clark, p. 41)

Solution BRP-109.
(a) A collective risk model is a distribution for which frequency and severity is explicitly recognized. The model assumes that the severity of loss (random variable x) has a given distribution, and the aggregate loss is the sum of n of these severities, where n is also a random variable.
(b) A more advanced collective risk model would be needed for situations involving **(1)** continuous functions and **(2)** higher expected frequencies.
(c) Unlike a recursive model, most aggregate loss models allow for more than a single severity distribution to be used. (Clark, p. 41)

Problem BRP-110. ***Fill in the blanks*** (Clark, p. 42): The aggregate distribution may be evaluated using _________ or ____________ methods. Certain __________ methods can provide very close approximations to the theoretical distribution, with efficient computer time.

Solution BRP-110. The aggregate distribution may be evaluated using **simulation** or **numerical** methods. Certain **numerical** methods can provide very close approximations to the theoretical distribution, with efficient computer time. (Clark, p. 42)

Problem BRP-111. Let y be the loss amount and E[y] be the expected value of the loss amount. Let G(y) be the cumulative aggregate distribution function and φ(y) be the excess charge factor at a given y. What is the formula for $E[y \mid L < y < U]$, the expected aggregate loss in a range with lower endpoint L and upper endpoint U? (Clark, p. 42)

Solution BRP-111. E[y │L < y < U] = (E[y]*(φ(L)-φ(U)) + L*(1-G(L)) – U*(1-G(U)))/(G(U)-G(L)).

Problem BRP-112. You are creating an aggregate distribution model for losses whose expected value is 500,000. You seek to find the expected value of losses within the range from 100,000 to 600,000, which is the layer covered by a reinsurance treaty (which offers coverage of 500,000 in excess of 100,000). You know that the aggregate cumulative distribution function G(y) is 0.25 at 100,000 and 0.89 at 600,000. The excess charge factors for the aggregate distribution function are 0.84 at 100,000 and 0.34 at 600,000. What is the expected value of losses within the layer covered by the reinsurance treaty?

Solution BRP-112. We seek to find E[y │100,000 < y < 600,000]. We use the formula E[y │L < y < U] = (E[y]*(φ(L)-φ(U)) + L*(1-G(L)) – U*(1-G(U)))/(G(U)-G(L)) for U = 600,000 and L = 100,000.

Thus, we seek to find E[y │100,000 < y < 600,000] = (E[y]*(φ(100,000)-φ(600,000)) + 100000*(1-G(100,000)) – 600000*(1-G(600,000)))/(G(600,000)-G(100,000))

We are given that G(100,000) = 0.25, G(600,000) = 0.89, φ(100,000) = 0.84, φ(600,000) = 0.34, and E[y] = 500000.

Thus, E[y │100,000 < y < 600,000] = (500000*(0.84-0.34) + 100000*(1-0.25) - 600000*(1-0.89))/(0.89-0.25) = **404,687.50**.

Problem BRP-113. ***Fill in the blanks*** (Clark, p. 42): The results of an aggregate distribution model are particularly important on "pure" ________ of ___________ covers, such as __________ treaties, which cover losses in excess of a set ____________ or _____________. The collective risk model is generally the best way to price these treaties.

Solution BRP-113. The results of an aggregate distribution model are particularly important on "pure" **excess** of **aggregate** covers, such as **Stop Loss** treaties, which cover losses in excess of a set **loss amount** or **loss ratio**. The collective risk model is generally the best way to price these treaties. (Clark, p. 42)

Problem BRP-114. (a) According to Clark (p. 42), what assumption do most collective risk models make regarding loss occurrences?

(b) Clark (p. 42) distinguishes between "process variance" and "parameter variance". Define each term.

(c) In a collective risk model, which of these – process variance or parameter variance – is always reflected by the aggregate distribution, and which might not be?

Solution BRP-114. (a) Most collective risk models assume that loss occurrences are independent of one another. This may or may not be true in reality.

(b) "Process variance" is "the random fluctuation of actual results about the expected value" (Clark, p. 42). "Parameter variance" can also be referred to as "model risk" and is uncertainty about whether the model's own design and parameters are appropriate for describing the situation in question (in Clark's words, "whether you are in the right model").

(c) The aggregate distribution of a collective risk model always reflects process variance, but not necessarily parameter variance.

Problem BRP-115. Describe two approaches that Clark (p. 42) recommends using to overcome the "black box" mentality – the assumption that results must be right because of the accuracy of the computer – when using aggregate distribution models.

Solution BRP-115. Clark's recommendations are the following:
1. Whenever possible, more than one set of results should be produced, as a check on the sensitivity of the answer to the starting assumptions.
2. Some basic statistics, such as the coefficient of variation and percentiles, should be compared to the empirical data for reasonability. (Clark, p. 42)

Problem BRP-116. *Fill in the blanks* (Clark, p. 42): Some collective risk models use _______ methods with a large error term for scenarios of __________. The expected _________ should be given in the output of the model.

Solution BRP-116. Some collective risk models use **numerical** methods with a large error term for scenarios of **low frequency**. The expected **error term** should be given in the output of the model. (Clark, p. 42)

Problem BRP-117.
(a) When there is a property catastrophe reinsurance treaty, do other reinsurance agreements (e.g., surplus share treaties, per-risk excess treaties, facultative certificates) typically inure to the benefit of the property catastrophe treaty, or does the property catastrophe treaty typically inure to the benefit of the other reinsurance?
(b) *Fill in the blank* (Clark, p. 43): The limit in a property catastrophe reinsurance treaty is defined in excess of a(n) _____________.

Solution BRP-117.
(a) Other reinsurance agreements typically inure to the benefit of the property catastrophe treaty.
(b) The limit in a property catastrophe reinsurance treaty is defined in excess of a **total loss amount**. (Clark, p. 43)

Problem BRP-118. You have the following information about a catastrophe excess-of-loss reinsurance treaty for the annual term encompassing the entire year 2022:
Annual premium: $4,000,000
Occurrence limit: $50,000,000
Date of loss: September 1, 2022
Loss amount: $35,000,000
Reinstatement provision: 120%

(a) Calculate the reinstatement premium after the loss if the reinstatement provision is pro rata as to amount, but not pro rata as to time.

(b) Calculate the reinstatement premium after the loss if the reinstatement provision is pro rata as to amount *and* pro rata as to time.

(c) Why are most reinstatement premiums for catastrophe excess-of-loss treaties *not* pro rata as to time? (See Clark, p. 43.)

Solution BRP-118. (a) If the reinstatement provision is pro rata as to amount, but not pro rata as to time, then we only need to consider the fraction of the annual premium corresponding to the proportion of the loss amount to the occurrence limit, multiplied by the percentage in the reinstatement provision.
Reinstatement premium = ($35,000,000/$50,000,000)*$4,000,000*120% = **$3,360,000.**

(b) If the reinstatement provision is pro rata as to amount and to time, then a further reduction of the reinstatement premium is needed to account for the time during which the new coverage will be effective - i.e., from September 1, 2022, until the end of 2022, or 4/12 = 1/3 years. Thus, the reinstatement premium is $3,360,000*(1/3) = **$1,120,000.**

(c) Most reinstatement premiums for catastrophe excess-of-loss treaties are not pro rata as to time because many catastrophes, such as hurricanes, occur seasonally, so a pro rata approach to time does not take into account the actual exposure to risk during the remainder of the treaty period.

Problem BRP-119. If the annual premium for a catastrophe reinsurance treaty is $600,000, and the treaty has a payback period of 10 years, how large of a single total loss amount would the treaty be expected to cover every 10 years? (See Clark, p. 44)

Solution BRP-119. A payback period of 10 years means that the treaty is expected to cover an amount of loss equal to 10*(Annual Premium) every 10 years. In this case, 10*(Annual Premium) = a **$6,000,000 loss every 10 years.**

Problem BRP-120. What are four main components of typical catastrophe models used for pricing property catastrophe reinsurance treaties? (Clark, p. 44)

Solution BRP-120. The following are four main components of typical catastrophe models:
1. Event sets that simulate the covered hazards (such as hurricanes or earthquakes).
2. Calculation of local event intensity for each property within a portfolio.
3. Estimation of damage for each property within a portfolio impacted by a given event.
4. Insured loss estimates based on policies written by the ceding company.

Problem BRP-121. Identify and briefly describe four types of information required for a catastrophe model used for pricing property catastrophe reinsurance treaties (Clark, pp. 44-45).

Solution BRP-121. The following are the four types of information required:

1. Measure of exposure: Examples include insured values, construction types, and occupancies, as well as attachment points for excess contracts.
2. Geographical information: Property information, converted into latitude and longitude coordinates, or aggregated insured-value information by zip code or state (which is less precise).
3. Terms of insurance policies: Deductibles, coinsurance provisions, etc., of the original insurance policies.
4. Details of inuring reinsurance: Any features of reinsurance treaties that inure to the benefit of the catastrophe treaties – e.g., occurrence caps or loss corridors – that affect the catastrophe exposure. (Clark, pp. 44-45)

Problem BRP-122. ***Fill in the blanks*** (Clark, p. 45): The output of a catastrophe model is a _________ of possible losses on the subject business. The expected amount in the treaty layer, usually referred to as the _____________, can be calculated, along with its ____________, and can be used as a starting point for a ____________ on the cover.

Solution BRP-122. The output of a catastrophe model is a **distribution** of possible losses on the subject business. The expected amount in the treaty layer, usually referred to as the **average annual loss (AAL)**, can be calculated, along with its **standard deviation**, and can be used as a starting point for a **loss cost** on the cover. (Clark, p. 45)

Problem BRP-123.
(a) What does an Occurrence Exceedance Probability (OEP) curve represent?
(b) What does an Annual Exceedance Probability (AEP) curve represent? (Clark, p. 45)

Solution BRP-123.
(a) The OEP curve represents the probability that at least one event during the year will exceed a given loss amount.
(b) The AEP curve represents the probability that the total of all modeled events in a single year exceeds a given loss amount. (Clark, p. 45)

Problem BRP-124. What two exposures associated with earthquakes might need to be considered in pricing a catastrophe reinsurance treaty, even if earthquake coverage is not sold by the ceding company? (Clark, p. 45)

Solution BRP-124.
1. Workers' compensation exposure: Workers' compensation losses (for a ceding company that writes this line and a reinsurer that reinsures it) could be substantial if an earthquake occurs during standard working hours.
2. "Fire following" an earthquake: If the ceding company covers fire losses, there may still be exposure related to an earthquake, even if direct earthquake damage is excluded. (Clark, p. 45)

Problem BRP-125. Why is it important for a pricing analysis of a property catastrophe reinsurance treaty to examine the proportion of policyholders purchasing replacement-cost coverage instead of actual cash value? (Clark, p. 45)

Solution BRP-125. After a major catastrophe event, costs borne by the primary insurer may rise due to increased demand for materials and labor. This would particularly affect policies with a replacement-cost basis of coverage. (Clark, p. 45)

Problem BRP-126.
(a) For which of the following coverage bases for a catastrophe reinsurance treaty – risks attaching or losses occurring – is it possible for a reinsurer to pay twice on the same loss event?
(b) Provide a numerical example of how such double payment for the same loss event could occur.
(c) What provision in a reinsurance treaty can prevent this double payment and how? (Clark, pp. 45-46)

Solution BRP-126.
(a) It is possible for a catastrophe reinsurance treaty to pay twice on the same loss event under a **"risks attaching"** coverage basis.
(b) Suppose a catastrophe reinsurance treaty covers a layer of $5,000,000 and renews on January 1 of each year. There is a catastrophe event inflicting a total loss amount of $10,000,000, and half of the losses pertain to policies that took effect before January 1 of the given year, while the other half of the losses pertain to policies that took effect after January 1. If two reinsurance treaties with the same reinsurer and with otherwise identical terms are effective during the two consecutive years, the "risks attaching" coverage basis might require the reinsurer to pay $5,000,000 for the losses on policies that incepted before January 1, and another $5,000,000 for the losses on policies that incepted after January 1.
(c) An **"interlocking clause"** can prevent double payment by equitably apportioning losses that may be covered under more than one contract. (Clark, pp. 45-46)

Problem BRP-127.
(a) What two characteristics are common to most "finite risk" reinsurance covers?
(b) Give an example of each of these features. (Clark, p. 46)

Solution BRP-127.
(a) The following characteristics are common to most "finite risk" reinsurance covers:
1. Multiple-year features.
2. Loss-sensitive features such as profit commissions and additional premium formulas. (Clark, p. 46)
(b) Example of multiple-year feature: A provision that applies to a three-year period but is cancellable after the first or second year only if premium to date exceeds the loss payments.
Example of loss-sensitive features: A profit provision that returns 80% of the premium if the contract is loss-free for three years, but the contract charges a high annual premium upfront. (Clark, p. 46)

Problem BRP-128. What are the two conditions under which a ceding company can consider an agreement to be reinsurance? (Clark, p. 46)

Solution BRP-128.
Condition 1. The reinsurer must assume significant reinsurance risk under the reinsured portions of the underlying insurance agreements.
Condition 2. It must be reasonably possible for the reinsurer to realize a significant loss from the transaction. (Clark, p. 46)

Problem BRP-129. There is a "finite risk" reinsurance agreement which has a \$5,000,000 annual premium and covers an occurrence limit of \$15,000,000. (The rate on line, the ratio of premium to the full possible loss on the contract, is therefore 33.333333%.) The profit commission on this agreement is 70% after a 15% margin on the annual premium. The premium is also adjustable, and, in the event of a loss, the additional premium is equal to 60% of the amount of (Loss + Margin – Annual Premium).

(a) Convert this agreement into an economically equivalent (for the reinsurer) traditional reinsurance agreement, where there is a fixed premium and no profit commissions exist.
(b) What is the rate on line of the traditional reinsurance agreement in part (a)? (See Clark, p. 47.)

Solution BRP-129. We first consider the annual results of "finite risk" reinsurance agreement under the scenarios of no losses and one full loss.

	Loss-Free Scenario	**One Full Loss**
(a) Premium	\$5,000,000	\$5,000,000
(b) Loss	\$0	\$15,000,000
(c) Margin = (a)*0.15	\$750,000	\$750,000
(d) Profit Commission = Max(0, 0.7*((a)-(b)-(c)))	\$2,975,000	\$0
(e) Additional Premium = Max(0, 0.6*((b)+(c)-(a)))	\$0	\$6,450,000
(f) Underwriting Result for Reinsurer = (a)-(b)-(d)+(e)	\$2,025,000	-\$3,550,000

An economically equivalent contract would also produce an underwriting result of \$2,025,000 for the reinsurer under a loss-free scenario and a result of -\$3,550,000 if there was one full loss. The way to achieve this would be to charge a premium of \$2,025,000 (which would become profit in entirety if no loss occurred) and to cover a maximum loss of \$2,025,000 + \$3,550,000 = \$5,575,000.

The resulting traditional reinsurance agreement would look as follows:

	Loss-Free Scenario	One Full Loss
(a) Premium	$2,025,000	$2,025,000
(b) Loss	$0	$5,575,000
(c) Underwriting Result for Reinsurer = (a)-(b)	$2,025,000	-$3,550,000

(b) The rate on line is equal to the ratio of premium to the full possible loss on the contract, which, in the case of the traditional reinsurance agreement from part (a), would be $2,025,000/$5,575,000 = **81/223 = 36.32286996%**.

Problem BRP-130.
(a) According to Clark, what time horizon is it best to use to estimate the different possible outcomes when a reinsurance treaty has complicated reinstatement provisions, carryforward provisions, and fluctuations in additional premium and profit commissions?
(b) In such situations, what simplifying assumption can allow the assignment of probabilities to each scenario using a Poisson or other distribution? (Clark, p. 48)

Solution BRP-130.
(a) Clark recommends a **one-year time horizon**.
(b) The simplifying assumption is that any penetrations into the reinsurance layer will exhaust the full limit. (Clark, p. 48)

Problem BRP-131.
(a) When a finite reinsurance agreement includes contingent additional premium that is charged if there are ceded losses, what additional consideration must the reinsurer now take into account?
(b) What document should be carefully reviewed by the reinsurer in light of this additional consideration? (Clark, p. 48)

Solution BRP-131.
(a) The reinsurer would need to take into account the **ceding company's credit risk**, because the ceding company might not be able to make the payment of additional premium.
(b) The reinsurer should carefully review the **ceding company's annual statement.** (Clark, p. 48)

Problem BRP-132.
(a) Give an example of an expense that applies to primary insurers but not to reinsurers.
(b) What are the three general types of reinsurer expenses? (Clark, p. 49)

Solution BRP-132.
(a) Premium tax is an expense that applies to primary insurers but not to reinsurers.
(b) The three general types of reinsurer expenses are the following:
1. Expenses varying with premium
2. Expenses varying with losses
3. Fixed expenses

Problem BRP-133. Classify each of the following under one of the three categories below:
1. Expenses varying with premium
2. Expenses varying with losses
3. Fixed expenses
(See Clark, p. 49.)

(a) Brokerage fees
(b) Salaries
(c) Federal excise tax
(d) Reinsurer's unallocated loss-adjustment expenses
(e) Ceding commission paid to the reinsured
(f) Real estate
(g) Underwriting and claim-audit expenses

Solution BRP-133.
(a) Brokerage fees - **1. Expenses varying with premium**
(b) Salaries **- 3. Fixed expenses**
(c) Federal excise tax - **1. Expenses varying with premium**
(d) Reinsurer's unallocated loss-adjustment expenses **- 2. Expenses varying with losses**
(e) Ceding commission paid to the reinsured - **1. Expenses varying with premium**
(f) Real estate **- 3. Fixed expenses**
(g) Underwriting and claim-audit expenses **- 3. Fixed expenses**

Problem BRP-134. Given that a loss cost of $500,000, fixed expenses of $23,000, an unallocated loss-adjustment-expense (ULAE) ratio of 4%, and a variable expense ratio of 30% were estimated for a given reinsurance treaty, what is the premium using the formula provided by Clark on p. 50?

Solution BRP-134. The formula provided by Clark is
Premium = [Loss Cost*(1+ULAE) + Fixed Expense]/(1-Variable Expense %), which in this case becomes Premium = (500000*(1+0.04) + 23000)/(1-0.3) = **$775,714.29**.

Problem BRP-135. ***Fill in the blanks*** (Clark, p. 50): When pricing a reinsurance treaty, in addition to considering loss costs, loss-adjustment expenses, and other fixed and variable expenses, consideration must also be given to the _______ and ________ elements of the contracts. The ___________ for the treaty need to be estimated, including premium and loss payments and any adjustable features.

Solution BRP-135. When pricing a reinsurance treaty, in addition to considering loss costs, loss-adjustment expenses, and other fixed and variable expenses, consideration must also be given to the **timing** and **risk** elements of the contracts. The **cash flows** for the treaty need to be estimated, including premium and loss payments and any adjustable features.